UNIT – II

INFINITE IMPULSE RESPONSE DIGITAL FILTERS

Syllabus: Review of design of analog butterworth and chebyshev filter, frequency transformation in analog domain, Design of IIR digital filter using impulse invariance Technique, Design of digital filters using Bilinear Transform, Pre warping, Realization using direct, Cascade & parallel forms

Review of Design of Analog Butterworth & Chebyshev Filter

Analog filter design is well-developed. All techniques are based on taking an analog filter and converting it to an digital filter.

Thus the design of IIR filter involves design of digital filter in analog domain and transforming the design into the digital domain.

Sh- system function describing analog

Impulse response of these filter coefficients is related to $H_a(s)$ by laplace transform

$$H_a(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

$H(s)$ can be described by linear constant coefficient difference eqn

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt}$$

Analog filter with $H(s)$ is stable if all its poles lie in the left half of s-plane.

To convert analog to digital domain the technique should possess the following properties

1) The $j\Omega$ axis in s-plane should map on to the unit circle in the z-plane.

2) The left half of s-plane should map into the inside of unit circle in the z-domain to convert a stable analog filter into a stable digital filter

IIR filter design by ~~at~~ impulse Invariant Technique

Desired impulse response of the digital filter is obtained by uniformly Sampling the impulse response of the equivalent analog filter

$$h(n) = h_a(nT) \quad \text{—— (1)}$$

T → sampling interval

$$H_a(s) = \sum_{i=1}^{M} \frac{A_i}{s-P_i} \quad \text{—— (2)}$$

By ~~inverse~~ taking inverse

$$h_a(t) = \sum_{i=1}^{M} A_i e^{P_i t} u_a(t) \quad \text{—— (3)}$$

$u_a(t)$ — unit step function in continous time.

$h(n)$ of digital filter is obtained by uniformly sampling $h_a(t)$

$$h(n) = h_a(nT) = \sum_{i=1}^{M} A_i e^{P_i n t} u_a(nt) \quad \text{—— (4)}$$

System Response of digital system can be obtained by taking Z-transform

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

using eqn (4)
$$H(z) = \sum_{n=0}^{\infty} \left[\sum_{i=1}^{M} A_i e^{P_i nT} u_a(nT) \right] z^{-n} \quad \text{—— (5)}$$

Compare (25) (6).

Mapping formula for IIT is

$$\boxed{\frac{1}{s-p_i} \rightarrow \frac{1}{1-e^{p_i T} z^{-1}}} \quad \text{--- (7)}$$

eqn (7) shows analog pole at $s = p_i$ is mapped into a digital pole at $z = e^{p_i T}$

Analog poles and Digital poles are related by $z = e^{sT}$ --- (8)

Substitute $s = \sigma + j\Omega$, $z = re^{j\omega}$

$$re^{j\omega} = e^{\sigma T} e^{j\Omega T}$$

$$r = e^{\sigma T}$$

$$\boxed{\omega = \Omega T} \quad \text{--- (9)}$$

Mapping $\omega = \Omega T$ implies that the interval $-\pi/T \leq \Omega \leq \pi/T$ maps into the corresponding values of $-\pi \leq \omega \leq \pi$

Properties of Impulse Invariant Transformation

$$\frac{1}{(s+s_i)^m} \longrightarrow \frac{(-1)^{m-1}}{(m-1)!} \frac{d^{m-1}}{ds^{m-1}} \left[\frac{1}{1-e^{-sT} z^{-1}}\right]_{s \to s_i}$$

$$\frac{s+a}{\ldots} \longrightarrow 1-e^{-aT}(\cos bT) z^{-1}$$

Pblm

Convert Analog filter into a digital filter whose system function is

$$H(s) = \frac{s+0.2}{(s+0.2)^2+9}$$

Use impulse invariant technique. Assume $T = 1$ sec

Soln

System Response of analog filter is of the standard form $H(s) = \frac{s+a}{(s+a)^2+b^2}$

$a = 0.2, \quad b = 3$

Using Impulse invariant technique property

$$\frac{s+a}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$H(z) = \frac{1-e^{-0.2T}(\cos 3T)z^{-1}}{1-2e^{-0.2T}(\cos 3T)z^{-1}+e^{-2(0.2)T}z^{-2}}$$

$T = 1$ sec

$$\frac{1-e^{-0.2}(\cos 3)z^{-1}}{1-2e^{-0.2}\cos 3\, z^{-1}+e^{-0.4}z^{-2}}$$

$$= \frac{1-(0.8187)(-0.99)z^{-1}}{1-2(0.8187)(-0.99)z^{-1}+0.6703\,z^{-2}}$$

Ans $$\boxed{H(z) = \frac{1+(0.8105)z^{-1}}{1+1.6210\,z^{-1}+0.6703\,z^{-2}}}$$

IIR filter design by Bilinear Transformation (BLT)

IIT technique is suitable for LPF, BPF design. This technique is not suitable for HPF, BRF. This limitation is overcome in the mapping technique called Bilinear Transformation

Also known as one-one mapping.

Bilinear Transformation is a conformal mapping that transforms $j\Omega$ axis into the unit circle in the Z-plane only once.

Relation b/w analog and digital frequencies

$$\boxed{\Omega = \frac{2}{T} \tan \frac{\omega}{2}}$$

(or)

$$\omega = 2 \tan^{-1} \frac{\Omega T}{2}$$

BLT Transformation : $\boxed{s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}}$

(or)

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

Frequency warping :

$\omega = 2 \tan^{-1} \frac{\Omega T}{2}$ $\quad \Omega = \frac{2}{T} \tan \frac{\omega}{2}$

Entire range in Ω is mapped only once into the range $-\pi \leq \omega \leq \pi$.

For hig frequencies mapping b/w ω, Ω becomes non linear, distortion is introduced in the frequency scale of the analog filter.

The mapping is nonlinear & lower frequencies in analog domain are expanded in digital domain, whereas higher frequencies are compressed. This is due to the nonlinearity of tangent function and is usually called frequency warping

Pre warping

warping can be eliminated by prewarping analog filter. It can be done by finding pre warping frequencies using the formula

$$\Omega_p = \frac{2}{T} \tan \frac{\omega}{2}$$

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2}$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2}$$

Pblm Convert analog filter with system function

$$H(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

into a digital IIR filter using Bilinear Transformation. Digital filter should have a resonant frequency of $\omega_r = \pi/4$

Soln

From the system function $\Omega_c = 3$

$$H(s) = \frac{s+a}{(s+a)^2+\Omega_c^2}$$

T can be determined by using the eqn $\Omega = \frac{2}{T} \tan\frac{\omega}{2}$

$$\Omega_c = \frac{2}{T} \tan\frac{\omega_r}{2}$$

$$T = \frac{2}{\Omega_c} \tan\frac{\omega_r}{2} = \frac{2}{3} \tan\frac{\pi/4}{2}$$

$$= \frac{2}{3} \tan \pi/8$$

$$T = 0.276 \text{ sec}$$

Using Bilinear Transformation

$$H(z) = H(s) \Big/ s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$H(z) = \frac{\frac{2}{T}\frac{z-1}{z+1} + 0.1}{\left[\frac{2}{T}\frac{(z-1)}{(z+1)} + 0.1\right]^2 + 9}$$

$T = 0.276$ sec

$$H(z) = \frac{1 + 0.027z^{-1} - 0.973z^{-2}}{8.572 - 11.84z^{-1} + 8.177z^{-2}}$$

BUTTERWORTH FILTER DESIGN

Pblm

Determine $H(z)$ for a butterworth filter satisfying the following constraints

$$\sqrt{0.5} \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le \pi/2$$

$$|H(e^{j\omega})| \le 0.2$$

with $T = 1$ sec. Apply Impulse invariant transformation

Soln

Given $\delta_1 = \sqrt{0.5} = 0.707$

$\delta_2 = 0.2$

$\omega_1 = \pi/2$

$\omega_2 = 3\pi/4$

Step 1: Determination of edge freqs of analog filter

$$\Omega_1 = \frac{\omega_1}{T} = \pi/2$$

$$\Omega_2 = \frac{\omega_2}{T} = 3\pi/4$$

$$\boxed{\frac{\Omega_2}{\Omega_1} = 1.5}$$

$$= \frac{1}{2}\frac{\log\left\{\frac{\left(\frac{1}{(0.2)^2}-1\right)}{\left(\frac{1}{(0.707)^2}-1\right)}\right\}}{\log(1.5)}$$

$$= \frac{1}{2}\frac{\log(24/11)}{\log(1.5)} = 3.91$$

Let $\boxed{N = 4}$

Step 3: Determination of 3-dB cutoff frequency

$$\Omega_c = \frac{\Omega_1}{\left[\left(\frac{1}{\delta_1^2}\right)-1\right]^{\frac{1}{2N}}}$$

$$= \frac{\pi/2}{\left[\left(\frac{1}{0.707^2}\right)-1\right]^{1/8}} = \pi/2$$

$$\boxed{\Omega_c = \pi/2}$$

Step 4: Determination of $H_a(s)$

Here $N = 4$ So $H(s) = \prod_{k=1}^{N/2} \frac{B_k \Omega_c^2}{s^2 + b_k\Omega_c s + C_k\Omega_c^2}$

$N = 2, 4, 6 \ldots$

$$= \prod_{k=1}^{2} \frac{B_k \Omega_c^2}{s^2 + b_k\Omega_c s + C_k\Omega_c^2}$$

$$= \frac{B_1 \Omega_c^2}{\ldots} \times \frac{B_2 \Omega_c^2}{s^2 + b_2\Omega_c s + C_2\Omega_c^2}$$

$K=1,2,\quad b_1 = 2\sin\left(\frac{\pi}{2\times 4}\right) = 2\sin \pi/8 = 0.76536$

$c_1 = 1$

$K=2,\quad b_2 = 2\sin\left(\frac{3\pi}{8}\right) = 1.84776.$

$c_2 = 1$

parameter B_k :-

$$A = \prod_{k=1}^{N/2} B_k \quad \text{for } N \text{ even}$$

Here $A=1$, so $\prod_{k=1}^{2} B_k = 1$

$B_1 B_2 = 1$

$\therefore \boxed{B_1 = 1, \quad B_2 = 1}$

Substitute coefficients & parameter in $H_a(s)$

$$\therefore H(s) = \frac{1(\pi/2)^2}{s^2 + 0.76536(\pi/2)s + 1(\pi/2)^2} \times \frac{1(\pi/2)^2}{s^2 + 1.8478(\pi/2)s + 1(\pi/2)^2}$$

$$= \frac{2.467}{s^2 + 1.2022s + 2.467} \times \frac{2.467}{s^2 + 2.9025s + 2.467}$$

Using partial fractions

$$\frac{2.467}{s^2+1.2022s+2.467}\,\frac{2.467}{s^2+2.9025s+2.467} = \frac{As+B}{s^2+1.2022s+2.467} + \frac{Cs+D}{s^2+2.9025s+2.467}$$

comparing two expressions,

$(2.467)(2.467) = (s^2 + 2.9025s + 2.467)(As+B)$

Comparing the coefficients of s^3, s^2, s & constant

$$6.086 = As^3 + Bs^2 + 2.9025As^2 + 2.9025Bs + 2.467As + 2.467B + Cs^3 + Ds^2 + 1.2022s^2C + 1.2022sD + 2.467Cs + 2.467D$$

s^3 coefficients, $A + C = 0$ — (1)

s^2 coefficients, $2.9025A + B + 1.2022C + D = 0$ — (2)

s coefficients, $2.9025B + 2.467A + 1.2022D + 2.467C = 0$ — (3)

Constants, $2.467B + 2.467D = 6.086$ — (4)

Solving eqn (1) (2) (3) (4)

$A = -1.4509$

$B = -1.7443$

$C = 1.4509$

$D = 4.2113$

Substituting A, B, C, D in H(s)

$$H(s) = \frac{-(1.4509s + 1.7443)}{s^2 + 1.2022s + 2.467} + \frac{1.4509s + 4.2113}{s^2 + 2.9025s + 2.467}$$

Let $H(s) = H_1(s) + H_2(s)$

Rearranging H(s) into standard form

$$H_1(s) = -\left(\frac{1.4509s + 1.7443}{s^2 + 1.2022s + 2.467}\right)$$

$$= -1.4509\left[\frac{s + 1.2022}{(s+0.601)^2 + (1.451)^2}\right]$$

$H_2(s)$ can be written as

$$H_2(s) = 1.4509\left[\frac{s+1.45}{(s+1.45)^2+(0.604)^2}\right] + 3.4903\left[\frac{0.604}{(s+1.45)^2+(0.604)^2}\right]$$

Step 5: Determination of $H(z)$

$\therefore$ IIT is used, so use the formula.

$$\frac{s+a}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+2e^{-aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \longrightarrow \frac{e^{-aT}\sin bT\, z^{-1}}{1-2e^{-aT}\cos bT z^{-1}+e^{-aT}z^{-2}}$$

$$H_1(z) = (-1.4509)\left[\frac{1-e^{-0.601T}(\cos 1.451T)z^{-1}}{1-2e^{-0.601T}(\cos 1.451T)z^{-1}+e^{-1.202T}z^{-2}}\right.$$

$$\left. -(0.601)\left[\frac{e^{-0.601T}(\sin 1.451T)z^{-1}}{1-2e^{-0.601T}(\cos 1.451T)z^{-1}+e^{-1.202T}z^{-2}}\right]\right]$$

&

$$H_2(z) = (1.4509)\,\frac{1-e^{-1.45T}(\cos 0.604T)z^{-1}}{1-2e^{-1.45T}(\cos 0.604T)z^{-1}+e^{-2aT}z^{-2}}$$

$$+\,3.4903\,\frac{e^{-1.45T}(\sin 0.604T)z^{-1}}{1-2e^{-1.45T}(\cos 0.604T)z^{-1}+e^{-2.9T}z^{-2}}$$

$T = 1$ sec

$H(z) = H_1(z) + H_2(z)$

$H(z) = -1.4509 - 0.\ldots z^{-1}$

Frequency Transformation in Analog Domain

Frequency transformation is used to design low pass filters with different pass band frequencies, high pass filters, Band pass filters, Band stop filters from a normalized low pass analog filter.

LPF to LPF

$s \rightarrow s/\Omega_c$ — Transformation used

Given : A normalized LPF

To design : To have LPF with different cutoff frequency Ω_c

LPF to HPF

Given : Normalized LPF

To design : HPF with cutoff freq Ω_c

Transformation $s \rightarrow \frac{\Omega_c}{s}$

LPF to BPF

Given : Normalized LPF

To design : BPF with cutoff freq Ω_l, Ω_u

transformation

$s \rightarrow \frac{s^2 + \Omega_l \Omega_u}{\quad}$

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1 (\Omega_u - \Omega_l)}$$

$$B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2 (\Omega_u - \Omega_l)}$$

(iv) LPF to BSF

Given : Normalized LPF

To have a BSF with cutoff frequencies, Ω_u, Ω_l

Transformation $S \rightarrow \frac{S(\Omega_u - \Omega_l)}{S^2 + \Omega_l \Omega_u}$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{\Omega_1 (\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u}$$

$$B = \frac{\Omega_2 (\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_l \Omega_u}$$

(v) Design of IIR filter using IIT

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$H(z)\Big|_{z=e^{sT}} = \sum_{n=0}^{\infty} h(n) e^{-sTn}$$

Relation b/w s-plane to z-plane.

Substitute $s = \sigma + j\Omega$

$$z = re^{j\omega}$$

$$re^{j\omega} = e^{(\sigma + j\Omega)T} = e^{\sigma T} e^{j\Omega T}$$

$$r = e^{\sigma T} \quad \text{——} \quad (1)$$

$$\omega = \Omega T \quad \text{——} \quad (2)$$

$H_a(s)$ is the system function of analog filter

$H_a(s)$ can be expressed in partial fraction form as

$$H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k}$$

$\{P_k\} \leftarrow$ Poles of analog filter

$\{C_k\} \leftarrow$ Coefficients in partial fraction expression

ILT of $H_a(s) \Rightarrow h_a(t) = \sum_{k=1}^{N} C_k e^{P_k t} \quad t \geq 0$

Sample $h_a(t)$ periodically at $t = nT$

$$h(n) = h_a(nT)$$

$$= \sum_{k=1}^{N} C_k e^{P_k nT}$$

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{n}$$

Sub $h(n)$ in $H(z)$

$$H(z) = \sum_{n=0}^{\infty} \sum_{k=1}^{N} C_k e^{P_k nT} z^{-n}$$

$$= \sum_{k=1}^{N} C_k \sum_{n=0}^{\infty} \left(e^{P_k T} z^{-1}\right)^n$$

Steps to design digital filter using IIT

1) For a given specification find Ha(S) transfer function of the filter.

2) Select sampling rate of digital filter T sec/samples

3) Express Ha(S) as sum of single pole filters

$$H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k}$$

4. Compute Z-transform of digital filter

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$$

For high sampling rate

$$H(z) = \sum_{k=1}^{N} \frac{T C_k}{1 - e^{P_k T} z^{-1}}$$

Steps to design digital filter using BLT

1) From the given specification, find Prewarping analog frequencies using the formula.

$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$

2) Using analog frequencies find H(S) of analog filter

3) Select sampling rate of digital filter.

T sec/sample

4) Substitute

CHEBYSHEV FILTERS

Chebyshev low pass filter has a magnitude response

$$|H(j\Omega)| = \frac{A}{\left[1+\varepsilon^2 C_N^2\left(\frac{\Omega}{\Omega_c}\right)\right]^{0.5}} \quad \text{--- (1)}$$

$A \leftarrow$ filter gain

$\varepsilon \leftarrow$ constant

$\Omega_c \leftarrow$ 3-dB cut-off freq

Chebyshev polynomial of I kind of N^{th} order $C_N(x)$ is given by

$$C_N(x) = \begin{cases} \cos(N\cos^{-1}x), & \text{for } |x| \le 1 \\ \cosh(N\cosh^{-1}x), & \text{for } |x| \ge 1 \end{cases} \quad \text{--- (2)}$$

Magnitude response of the Chebyshev filter is shown in fig. The magnitude response has equiripple pass band and maximally flat stop band.

By increasing order of filter 'N', chebyshev response approximates ideal response.

Phase response of chebyshev filter is more non linear than butterworth filter for a given filter length 'N'.

LPF Specifications

$$\delta_1 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le \omega_1$$

Substitute eqn ① in ③ and if $A=1$, we get

$$\delta_1^2 \leq \frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega_1}{\Omega_c}\right)} \leq 1$$

$$\frac{1}{1+\varepsilon^2 C_N^2\left(\frac{\Omega_2}{\Omega_c}\right)} \leq \delta_2^2$$

Assume $\Omega_c = \Omega_1$

$$C_N\left(\frac{\Omega_c}{\Omega_c}\right) = C_N(1) = 1$$

$$\therefore \delta_1^2 \leq \frac{1}{1+\varepsilon^2(1)} \leq 1 \quad \text{—— ⑤}$$

Assume equality in the eqn ⑤

$$\delta_1^2 = \frac{1}{1+\varepsilon^2}, \quad 1+\varepsilon^2 = \frac{1}{\delta_1^2}$$

$$\varepsilon^2 = \frac{1}{\delta_1^2} - 1$$

$$\boxed{\varepsilon = \sqrt{1/\delta_1^2 - 1}} \quad \text{—— ⑥}$$

Order of Analog filter Can be determined by eqn ④ ii

Assume $\Omega_c = \Omega_1$

Coefficients b_k, c_k are given by

$$b_k = 2 y_N \sin\left[\frac{(2k-1)\pi}{2N}\right]$$

$$c_k = y_N^2 + \cos^2\frac{(2k-1)\pi}{2N} \quad \text{—— ⑦}$$

Parameter y_N is given by

$$y_N = \frac{1}{2}\left\{\left[\sqrt{\frac{1}{\varepsilon^2}+1}+\frac{1}{\varepsilon}\right]^{1/N} - \left[\sqrt{\frac{1}{\varepsilon^2}+1}+\frac{1}{\varepsilon}\right]^{-1/N}\right\}$$

Parameter B_k can be obtained from

$$\boxed{\begin{aligned} \frac{A}{(1+\varepsilon^2)^{0.5}} &= \prod_{k=1}^{N/2} \frac{B_k}{C_k} \quad \text{for } N \text{ even} \\ A &= \prod_{k=0}^{\frac{N-1}{2}} \frac{B_k}{C_k} \quad \text{for } N \text{ odd.} \end{aligned}} \qquad (9)$$

System function $H(z)$ of equivalent digital filter is obtained from $H(s)$ using specified transformation technique IIT or BLT

Pblm Design a digital chebyshev filter to satisfy the constraints

$$0.707 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.2\pi$$
$$|H(e^{j\omega})| \le 0.1 \qquad 0.5\pi \le \omega \le \pi$$

using BLT, assume $T = 1$ sec.

Soln

Given : $\delta_1 = 0.707$ $\quad \omega_1 = 0.2\pi$

$\delta_2 = 0.1$ $\quad \omega_2 = 0.5\pi$

Step 1 : Determination of analog filter's digital frequencies

$$\Omega_c = \Omega_1 = \frac{2}{T} \tan \frac{\omega_1}{2}$$

$$\Omega_1 = \frac{2}{1} \tan \frac{0.2\pi}{2} = 0.6498$$

$$\Omega_2 = \frac{2}{T} \tan \frac{\omega_2}{2} = \frac{2}{1} \tan \frac{0.5\pi}{2} = 2$$

$$\boxed{\frac{\Omega_2}{\Omega_1} = 3.0779}$$

Step 2 : Determination of the order of the filter

$$\varepsilon = \sqrt{\left(\frac{1}{\delta_1^2} - 1\right)} = \sqrt{\left(\frac{1}{(0.707)^2}\right) - 1} = 1$$

order $\quad N \geq \dfrac{\cosh^{-1}\left\{\frac{1}{\varepsilon}\sqrt{\left[\frac{1}{\delta_2^2} - 1\right]}\right\}}{\cosh^{-1}\left(\Omega_2 / \Omega_1\right)}$

$$\geq \frac{\cosh^{-1}\left\{\frac{1}{1}\sqrt{\left(\frac{1}{0.1^2} - 1\right)}\right\}}{\cosh^{-1}\left(3.0779\right)}$$

$N \geq 1.669$ $\quad \boxed{N = 2}$

Step 3 : Determination of $H(s)$

For N even $\quad H(s) = \prod_{k=1}^{N/2} \dfrac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$

To find b_1, c_1 we need y_N parameter

$$y_N = \frac{1}{2}\left\{\left[\sqrt{\frac{1}{\varepsilon^2}+1}+\frac{1}{\varepsilon}\right]^{1/N} - \left[\sqrt{\frac{1}{\varepsilon^2}+1}+\frac{1}{\varepsilon}\right]^{-1/N}\right\}$$

$N=2$,

$$y_2 = \frac{1}{2}\left\{\left[\sqrt{\frac{1}{1}+1}+\frac{1}{1}\right]^{1/2} - \left[\sqrt{\frac{1}{1}+1}+\frac{1}{1}\right]^{-1/2}\right\}$$

$$= \frac{1}{2}\left\{\left[1.414+1\right]^{1/2} - \left[1.414+1\right]^{-1/2}\right\}$$

$$= 0.5\left[1.5537 - 0.6436\right]$$

$$\boxed{y_2 = 0.455}$$

$$b_K = 2y_N \sin\left[\frac{(2K-1)\pi}{2N}\right]$$

$K=1$ $\quad b_1 = 2y_2 \sin\left[\frac{(2(1)-1)\pi}{2(2)}\right]$

$$\boxed{b_1 = 0.6435}$$

$$c_K = y_N^2 + \cos^2\frac{(2K-1)\pi}{2N}$$

$K=1$, $\quad c_1 = y_2^2 + \cos^2\left[\frac{(2(1)-1)\pi}{2(2)}\right]$

$$\boxed{c_1 = 0.7070}$$

For N even

$$\prod_{K=1}^{N/2} \frac{B_K}{c_K} = \frac{A}{\sqrt{1+\varepsilon^2}}$$

$\frac{B_1}{c_1} = \frac{1}{1.414}$ $\qquad B_1 = c_1 \times 0.707$

System function. $H(s) = \dfrac{0.5\,(0.6498)^2}{s^2 + 0.6435\,(0.6498)s + 0.707\,(0.6498^2)}$

$$H(s) = \frac{0.2111}{s^2 + 0.4185 + 0.2985}$$

Step (iv) Determination of $H(z)$

using BLT $H(z) = H(s) \Big/ s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

$$= \frac{0.2111}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + 0.418\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.2985\right]}$$

Ans $H(z) = \dfrac{0.0411\,[1+z^{-1}]^2}{1 - 1.4418 z^{-1} + 0.6743 z^{-2}}$

Realization using DIRECT CASCADE & PARALLEL FORMS