EC 6502. Principles of DSP.

UNIT - V DSP APPLICATIONS

MULTIRATE SIGNAL PROCESSING:-

Decimation, Interpolation, sampling rate conversion by a rational factor.

Adaptive Filters:-

Introduction, Applications of Adaptive filtering to equalization.

# EC6502 Principles of Digital Signal Processing

## UNIT V Applications of DSP

Prepared by
S.Janakiraman
AP/ECE

## Chapter 5
## Sampling Rate Conversion

Content

- Introduction
- Decimation by a factor D
- Interpolation by a factor I
- Sampling rate conversion by a rational factor I/D

## Introduction

- In many practical applications of DSP, one is faced with the problem of changing the sampling rate of a signal.
- The process of converting a signal from a given rate to a different rate is called *sampling rate conversion*.
- Systems that employ multiple sampling rates in the processing of digital signals are called *multirate digital signal processing systems*.

## Introduction

- There are two general methods to accomplish the sampling rate conversion of a digital signal.
  - To pass the digital signal through a D/A converter, filter it if necessary, and then to resample the resulting analog signal at the desired rate;
  - To perform the sampling rate conversion entirely in the digital domain.
- The process of reducing the sampling rate by an integer factor *D* (downsampling by *D*) is called *decimation*.
- The process of increasing the sampling rate by an integer factor *I* (upsampling by *I*) is called *interpolation*.

## Decimation by a factor D

In downsampling by an integer factor $D>1$, every $D$-th samples of the input sequence are kept and others are removed:

$$x_d(n) = x(Dn)$$

$$\underset{f_s}{x(n)} \longrightarrow \boxed{\downarrow D} \longrightarrow \underset{\frac{f_s}{D}}{x_d(n)}$$

## Decimation by a factor D

- Relationship in time domain

$x(n)$ Input sequence

$p(n) = \sum_{k=-\infty}^{\infty} \delta(n-kD)$ Periodic train of impulses

$x_p(n) = x(n)p(n)$

$x_d(n) = x_p(Dn) = x(Dn)$ Output sequence

## Decimation by a factor D

- Relationship in frequency domain

$$X_p(e^{j\omega}) = \frac{1}{2\pi}\int_0^{2\pi} P(e^{j\theta})X(e^{j(\omega-\theta)})d\theta$$

$$p(n) = \frac{1}{D}\sum_{k=0}^{D-1} P(k)e^{j\frac{2\pi}{D}kn}, \quad P(k) = \sum_{n=0}^{D-1} p(n)e^{-j\frac{2\pi}{D}kn}$$

$$P(k) = \sum_{n=0}^{D-1}\left[\sum_{l=-\infty}^{\infty}\delta(n-lD)\right]e^{-j\frac{2\pi}{D}kn} = \sum_{n=0}^{D-1}\delta(n)e^{-j\frac{2\pi}{D}kn} = 1$$

$$P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} p(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty}\left[\frac{1}{D}\sum_{k=0}^{D-1} P(k)e^{j\frac{2\pi}{D}kn}\right]e^{-j\omega n}$$

$$= \frac{1}{D}\sum_{k=0}^{D-1}\sum_{n=-\infty}^{\infty} e^{j\frac{2\pi}{D}kn}e^{-j\omega n} = \frac{2\pi}{D}\sum_{k=0}^{D-1}\delta(\omega - \frac{2\pi}{D}k)$$

$$X_p(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} X(e^{j(\omega-k\omega_s)}) \qquad \omega_s = \frac{2\pi}{D}$$

$$X_d(e^{j\omega}) = \sum_{m=-\infty}^{\infty} x_d(m)e^{-j\omega m} = \sum_{m=-\infty}^{\infty} x_p(mD)e^{-j\omega m}$$

for $n = mD$

$$= \sum_{n=mD} x_p(n)e^{-j\omega\frac{n}{D}} = \sum_{n=-\infty}^{\infty} x_p(n)e^{-j\omega\frac{n}{D}} = X_p(e^{j\frac{\omega}{D}})$$

## Decimation by a factor D

- Using a digital low-pass filter to prevent aliasing

$$x(n) \longrightarrow \boxed{h(n)} \xrightarrow{x'(n)} \boxed{\downarrow D} \longrightarrow x_d(n)$$

$$H(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \frac{\pi}{D} \\ 0, & \text{otherwise} \end{cases}$$

## Interpolation by a factor I

In up-sampling by an integer factor $I > 1$, $I$-1 equidistant zeros-valued samples are inserted between each two consecutive samples of the input sequence. Then a digital low-pass filter is applied.

$$x_p(n) = \begin{cases} x(\frac{n}{I}), & n = 0, \pm I, \pm 2I \cdots \\ 0, & \text{otherwise} \end{cases}$$

$$\underset{f_s}{x(n)} \longrightarrow \boxed{\uparrow I} \xrightarrow{x_p(n)} \boxed{h(n)} \longrightarrow \underset{If_s}{x_I(n)}$$

## Interpolation by a factor I

Relationship in frequency domain

$x(n)$ Input sequence $\quad x_p(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-kI)$

$$X_p(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left(\sum_{k=-\infty}^{\infty} x(k)\delta(n-kI)\right)e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x(k)e^{-j\omega Ik} = X(e^{j\omega I})$$

$$H(e^{j\omega}) = \begin{cases} I, & 0 \le |\omega| \le \frac{\pi}{I} \\ 0, & \text{otherwise} \end{cases}$$

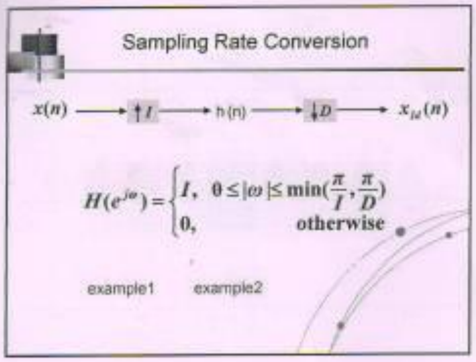
## Sampling rate conversion by a rational factor I/D

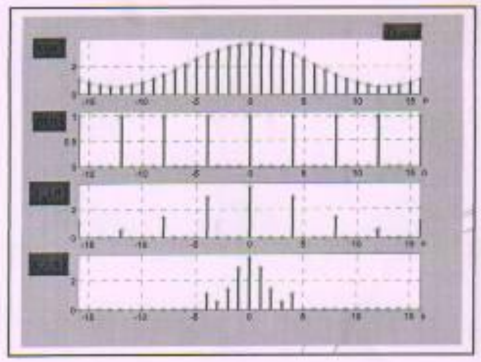
If $R = \frac{I}{D}$ is a rational number

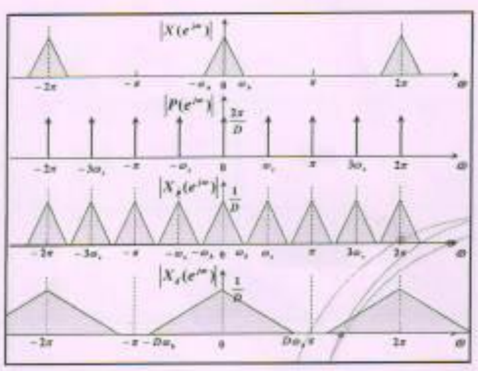
$$\underset{f_s}{x(n)} \longrightarrow \underbrace{\boxed{\uparrow I} \longrightarrow \boxed{h_1(n)}}_{\text{interpolation}} \xrightarrow[If_s]{x_I(n)} \underbrace{\boxed{h_2(n)} \longrightarrow \boxed{\downarrow D}}_{\text{decimation}} \longrightarrow \underset{\frac{I}{D}f_s}{x_{Id}(n)}$$

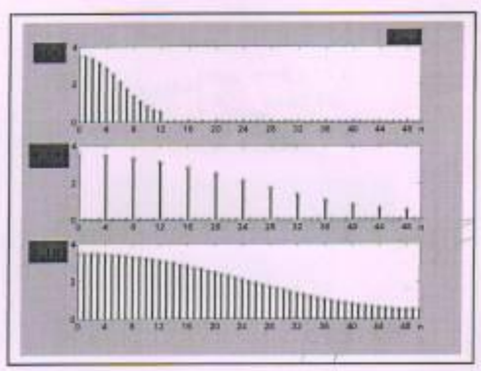
Sampling period

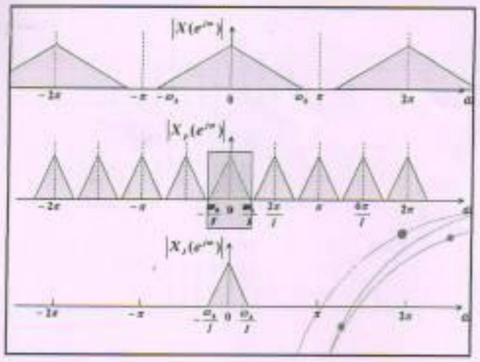
| | | | | |
|---|---|---|---|---|
| $T$ | $\frac{T}{I}$ | $\frac{T}{I}$ | $\frac{T}{I}$ | $\frac{DT}{I}$ |

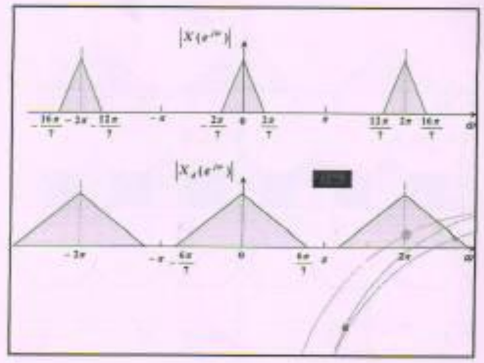

Sampling Rate Conversion
$x(n)$
$\uparrow I$
h(n)
$\downarrow D$
$x_{Id}(n)$
$H(e^{j\omega}) = \begin{cases} I, & 0 \le |\omega| \le \min(\frac{\pi}{I}, \frac{\pi}{D}) \\ 0, & \text{otherwise} \end{cases}$
example1
example2

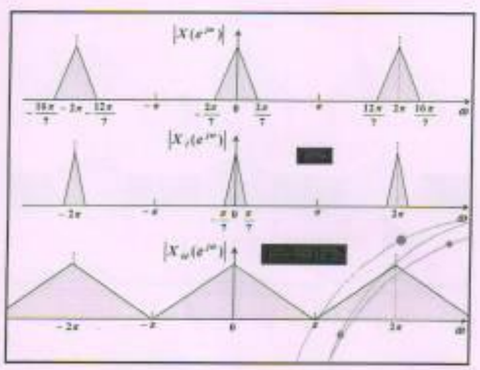












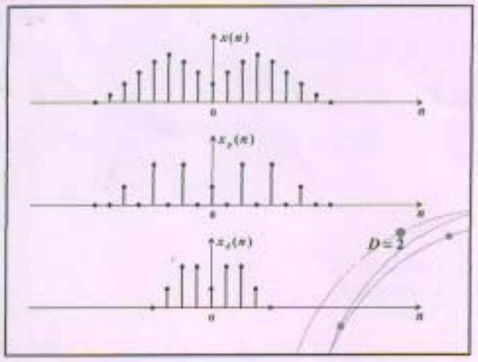
P137, No.26 in textbook

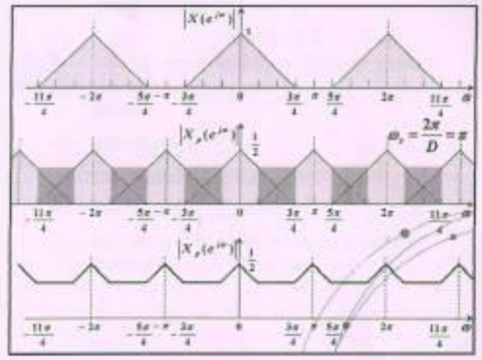
$$x_p(n)=\begin{cases} x(n), & n=0,\pm 2,\pm 4,\cdots \\ 0, & n=\pm 1,\pm 3,\cdots \end{cases}$$

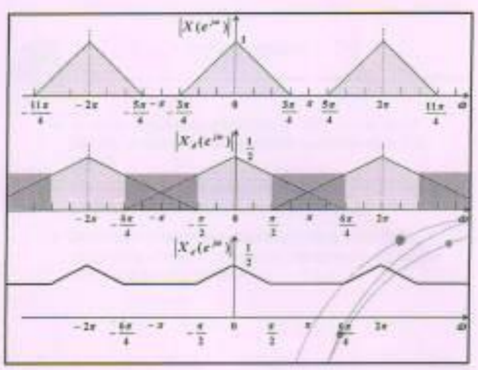
$$x_d(n)=x(2n)$$

$$X_p(e^{j\omega})=\frac{1}{D}\sum_{k=0}^{D-1}X(e^{j(\omega-k\omega_s)}),\quad \omega_s=\frac{2\pi}{D}$$

$$X_d(e^{j\omega})=X_p(e^{j\frac{\omega}{D}})$$


x(n)
D = 2


$\omega_s = \frac{2\pi}{D} = \pi$



## ADAPTIVE FILTERS:-

The most widely used adaptive filters based on the FIR filter with the least-mean-square (LMS) algorithm. These adaptive filters are relatively simple to design and implement. They are well understood with regard to stability, convergence speed, steady-state performance, and finite-precision effect.

### Introduction to Adaptive Filtering:-

An adaptive filter consists of two distinct parts - a digital filter to perform the desired filtering, and an adaptive algorithm to adjust the co-efficients (or) weights) of the filter.

A general form of adaptive filter is illustrated in figure 1.

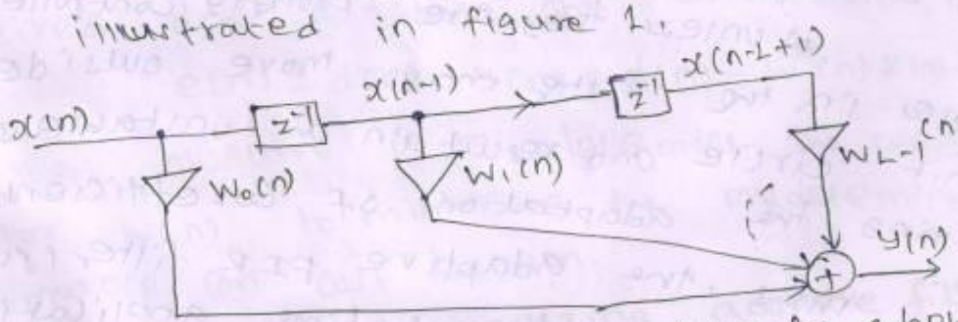



Fig 1: Block diagram of FIR filter for adaptive filtering.

where, $d(n)$ is a desired (or) (primary input) signal, $y(n)$ is the output of a digital filter driven by a reference input signal $x(n)$ and $e(n)$ is the error signal (It is

The adaptive algorithm adjust the fi[lter] coefficients to minimize the mean-squa[re] value of e(n). Therefore, the filter weig[hts] are updated so that the error is progres[sively] minimized on a sample-by sample bas[is].

In general, there are two types of digital filters that can be used for adapti[ve] filtering:-

FIR filter and IIR filter:-

FIR filter:-

It is always stable and can provide a linear phase response.

IIR filter:-

*) It involves both zeros and poles.

*) unless they are properly controlled, th[e] poles in the filter may move outside the unit circle and result in an unstable syste[m] during the adaptation of co-efficients.

Thus, the adaptive FIR filter is wide[ly] used for practical real-time applications.

The most widely used adaptive FIR filter is depicted in Fig 2.

The filte output signal is computed.

$$y(n) = \sum_{t=0}^{L-1} w_l(n)\, x(n-l) \rightarrow (1)$$

where the filter co-efficients $w_l(n)$ are time varying and updated by the adaptive algorithms

we define the input vector at time n as

$$x(n) = [x(n)\ x(n-1) \ldots x(n-L+1)]^T \rightarrow (2)$$

and the weight vector at time n as.

$$w(n) \equiv [w_0(n)\ w_1(n) \cdots w_{L-1}(n)]^T \rightarrow (3)$$

equ (1) can be expressed in vector formal.

$$y(n) = w^T(n) \cdot x(n) = x^T(n)\, w(n) \rightarrow (4)$$

The filte output $y(n)$ is compared with the desired $d(n)$ to obtain the error signal is

$$e(n) = d(n) - y(n) = d(n) - w^T(n)\, x(n) \rightarrow (5)$$

The objective is to detemine the weight vector $w(n)$ to minimize the predetemined performance (or) cost function.

Basic algorithms for realizing adaptive filters

1. Lms (Least mean square) algorithm, which is based on a gradient optimization for determining the co-efficients.
2. RLS (Recursive Least square) algorithms which include both direct form FIR and

Applications of Adaptive filtering:-

Adaptive filters have been widely used in Communication systems, Control system, and various other systems in which the statistical characteristics of the signal to be filtered are either unknown in some cases they are slowly time variant (or) non-stationary signals.

Some of the applications of adaptive filters includes:-

1. Adaptive antenna systems:- In which adaptive filters are used for beam steering and for providing nulls in the beam pattern to remove undesired interference

2. Digital Communication receivers:- In which adaptive filters are used to provide equalization of Intersymbol Interference (ISI) and for Channel identification.

3. Adaptive noise cancelling technique:- In which adaptive filters are used to estimate and eliminate a noise component in a desired signal.

4. System modelling:- In which adaptive filters are used as models to estimate the characteristics of an unknown system.

FIR filters are most widely used because it has only ... fixed

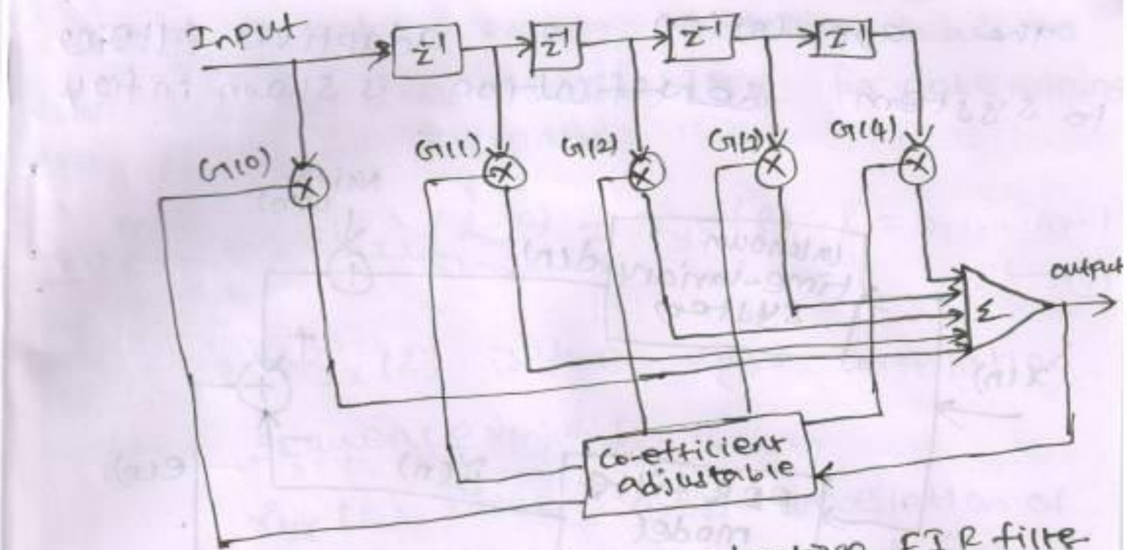



Fig. 3. Direct form adaptive FIR filter

The direct form and the lattice form FIR filter structure are used in adaptive filtering applications.

The direct form FIR filter structure with adjustable coefficient h(n) in shown in figure 3. An important consideration in the use of an adaptive filter is the criterion for optimizing the adjustable filter co-efficients.

Application of adaptive filters for system identification modeling:

In this application adaptive filters are used to identify unknown system (or) plant. The system is modeled by an FIR filter with 'M' adjustable co-efficients. Both the plant and model are excited by an input sequence x(n). Let y(n) denotes the

The application of adaptive filters to system identification is shown in fig 4

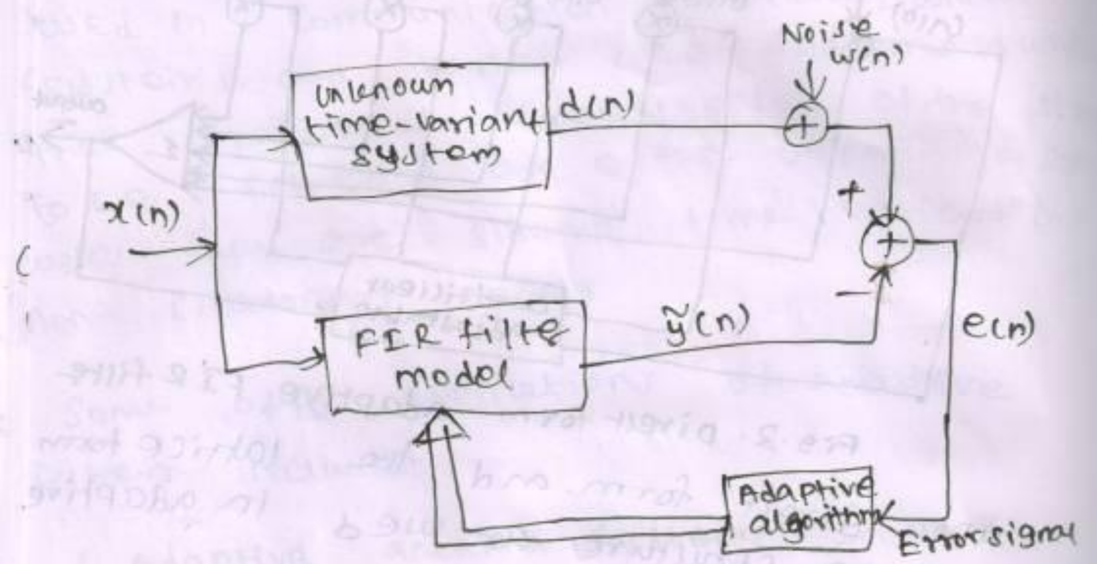



Fig 4: Applications of adaptive filtering to system identification.

Let ỹ(n) denote the output of the model and is given by,

$$\tilde{y}(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k) \longrightarrow (1)$$

The error in the output and desired response is;

$$e(n) = y(n) - \tilde{y}(n), \quad n = 0, 1, \ldots \longrightarrow (2)$$

The coefficients h(k) is used to minimize the error coefficients,

$$e_m = \sum_{n=0}^{N} \left[ y(n) - \sum_{k=0}^{M-1} h(k)\, x(n-k) \right]^2 \longrightarrow (3)$$

The Least squares criterion leads to the set of linear equations for determining the filter Co-efficients.

$$\sum_{k=0}^{M-1} h(k)\, \gamma_{xx}(l-k) = \gamma_{yx}(l), \quad l = 0,1 \ldots M-1 \quad \rightarrow (4)$$

where, $\gamma_{xx}(l)$ is the auto Correlation of the sequence $x(n)$ and

$\gamma_{yx}(l)$ is the cross correlation of the system output with input sequence

2) Application of Adaptive filters for Channel equalization:-

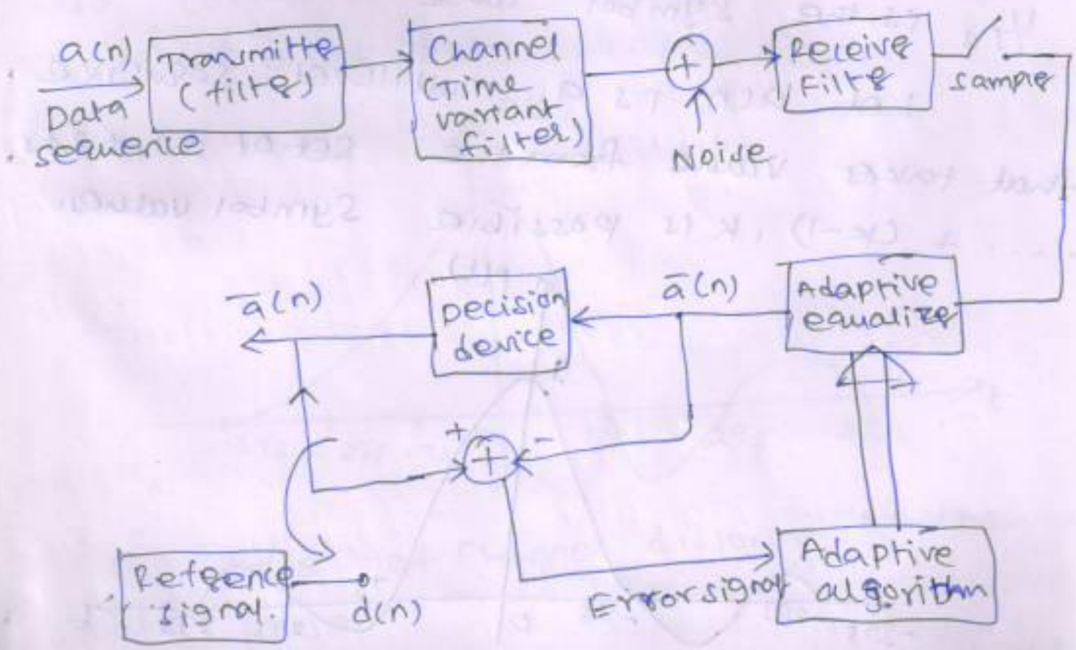



Fig. 5. Adaptive filtering to adaptive Channel equalization

Fig. 5 shows the block diagram of digital communication system in which an adaptive equalizer is used to compensate for the distortion caused by the transmission medium/channel.

The digital sequence of information symbols $a(n)$ is fed to the transmitting filter, the output is

$$s(t) = \sum_{k=0}^{\infty} a(k)\, p(t - kT_s) \rightarrow (1)$$

where, $p(t)$ is the impulse response of the filter at the transmitter and $T_s$ is the time interval between information symbols and $1/T_s$ is the symbol rate.

Let $a(n)$ is a multilevel sequence that takes value from the set of $\pm 1, \pm 3, \pm 5 \ldots \pm (k-1)$, $k$ is possible symbol values.

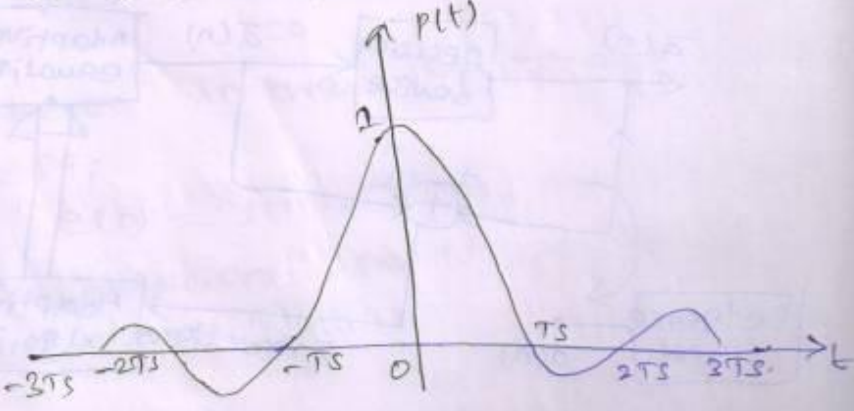



Fig. Pulse shape for digital transmission of symbols at a rate of $1/T_s$ symbols/second

The pulse p(t) have the Character as shown in fig. The p(t) has amplitude $p(0)=1$. at $t=0$ and $p(nT_s)=0$ at $t=nT_s$, $n=\pm1,\pm2$.

$\therefore$ successive pulses are transmitted sequentially for every $T_s$ seconds and do not interfere with one another when sampled at the time instants $t=nT_s$.

$$\boxed{\therefore a(n) = s(nT_s)} \longrightarrow (2)$$

The channel which is modelled as linear filter, distorts the pulse and cause intersymbol Interference. The distorted signal is also corrupted by additive noise, which is usually wide band signal.

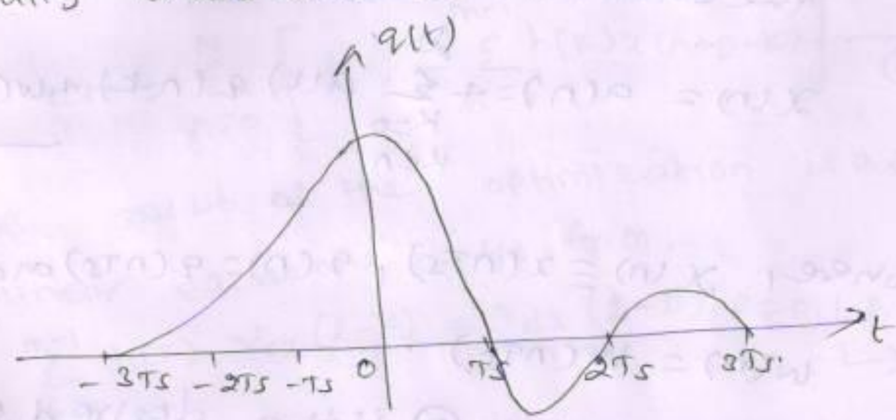



Fig. Effect of Channel distortion on the signal pulse.

*) samples of the received signal at the output of this filter reflect the presence ... and additive noise

The sampled output at the receiver is,

$$x(nT_s) = \sum_{k=0}^{\infty} a(k)\, q(nT_s - kT_s) + w(nT_s) \quad \longrightarrow (3)$$

$$= a(n)\, q(0) + \sum_{\substack{k=0 \\ k \neq n}}^{\infty} a(k)\, q(nT_s - kT_s) + w(nT_s) \quad \longrightarrow (4)$$

where, $w(t)$ is additive noise and $q(t)$ represents distorted pulse at the output of the receive file.

Assume that the sample $q(0)$ is normalized to unity by using automatic gain control in the receiver.

The sampled signal in eqn (4) is,

$$x(n) = a(n) + \sum_{\substack{k=0 \\ k \neq n}}^{\infty} a(k)\, q(n-k) + w(n) \quad \longrightarrow (5)$$

where, $x(n) \equiv x(nT_s)$, $q(n) = q(nT_s)$ and $w(n) = w(nT_s)$.

The term $a(n)$ in (5) is the desired sym at the $n^{th}$ sampling instant.

The term $\sum_{\substack{k=0 \\ k \neq n}}^{\infty} a(k)\, q(n-k)$ constitutes the ... are due to channel

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The term $a(n)$ in (5) is the desired symbol at the $n^{th}$ sampling instant.

The term $\sum_{\substack{k=0 \\ k \neq n}}^{\infty} a(k)\, q(n-k)$ Constitutes the ... are due to channel

The equalizer is an FIR filter with M adjustable co-efficients h(n).

It's output is expressed as:

$$\tilde{a}(n) = \sum_{k=0}^{M-1} h(k)x(n+D-k) \rightarrow ⑥$$

where, D is the ~~nominal~~ nominal delay in processing the signal through the filter and $\tilde{a}(n)$ represents an estimate of the nth information symbol.

By using least mean square error criterion the co-efficients h(k) is used to minimize the quantity.

$$e_m = \sum_{n=0}^{N} [d(n) - \tilde{a}(n)]^2$$

$$e_m = \sum_{n=0}^{N} \left[ d(n) - \sum_{k=0}^{m-1} h(k)x(n+D-k) \right]^2 \rightarrow ⑦$$

the result of the optimization is a set of linear equations of the form,

$$\sum_{k=0}^{m-1} h(k)\, \gamma_{xx}(l-k) = \gamma_{dx}(l-D), \quad l=0,1,2..m-1 \rightarrow ⑧$$

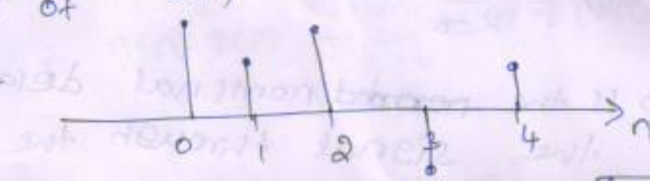
where,

$\gamma_{xx}(l) \rightarrow$ auto correlation of the sequence x(n)

$\gamma_{dx}(l) \rightarrow$ Cross correlation between the desired sequence d(n) [and the] received sequence x(n).

Problems in downsampling and upsampling:-

Example 1 :- plot the three fold upsampling signal y(n) for the signal x(n) shown in Fig (a) of




Solution

L=3


L-1 zero are added

Example 2 :- If $x(n) = \{1,2,4,-2,3,2,1\}$ find the two fold upsampling of the sequence

Solution

$$x(n) = \{1, 2, 4, -2, 3, 2, 1 \dots\}$$

$L = 2$, $L-1$ zeros are to be appended

w.k.t $y(n) = x(n/L)$

$$\therefore y(n) = \{1, 0, 2, 0, 4, 0, -2, 0, 3, 0, 2, 0 \dots$$

*) the zero values samples are inserted by upsampler are replaced with appropriate non-zero values using some type

Example 3 :- show that the upsample and down sampler are time variant systems.

Solution

Consider a factor L, upsampler is given by

$$y(n) = x\left(\frac{n}{L}\right) \rightarrow (1)$$

The output due to delayed input is,

$$y(n,k) = x\left(\frac{n-k}{L}\right) \rightarrow (2)$$

The delayed output is,

$$y(n-k) = x\left(\frac{n-k}{L}\right) \rightarrow (3)$$

from equation (2) and (3)

$$y(n,k) \neq y(n-k)$$

$\therefore$ Upsampling is a time variant system

Similarly for down sampler,

$$y(n) = x(nm)$$

$$y(n,k) = x(nm-k) \rightarrow (4)$$

$$y(n-k) = x(m(n-k)) \rightarrow (5)$$

$$y(n,k) \neq y(n-k)$$

$\therefore$ Upsampling and down sampling are time variant systems.

Example 4 :-

A signal $x(n)$ is given by $x(n) = \{0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3 \dots\}$

(i) obtain the decimated signal with a factor 2

(ii) obtain the interpolated signal with a factor 2.

Solution.

Given decimating factor $M = 2$

Interpolating factor $L = 2$

(i) Decimation by a factor 2. The given signal is

$$x(n) = \{0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3 \dots\}$$

The output signal after decimation by a factor 2.

$$y(n) = \{0, 2, 4, 6, 1, 3 \dots\}$$

(ii) Interpolation by a factor 2. The given signal is $x(n) = \{0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3 \dots\}$

The output signal after Interpolated by a factor 2

$\dots 4, 0, 5, 0, 6, 0, 0, 0$

## Example 5

Implement a two stage decimator for the following specifications:

sampling rate of the input signal = 20,000 Hz

M = 100

pass band = 0 to 40 Hz

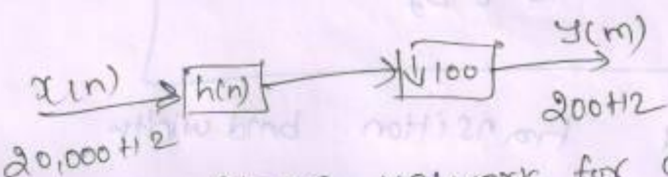
Transmission band = 40 Hz to 50 Hz

Pass band ripple = 0.01

Stop band ripple = 0.002.

## Solution

The implementation of the decimator for the given specifications is shown below




single stage Network for decimator for

2-stage decimator

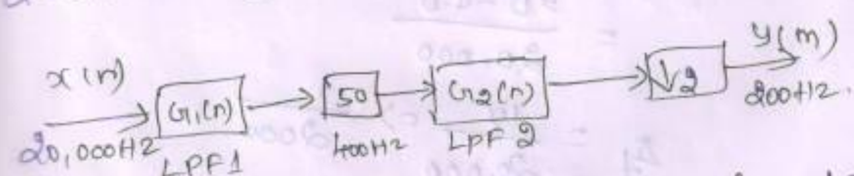



Fig. Two stage Network for decimator

The frequency response of LPF

Pass-band frequency, $F_p = 40$ Hz

Stop-band frequency, $F_s = 50$ Hz

pass-band ripple, $\delta_p = 0.01$

Stop-band ripple, $\delta_s = 0.002$

Transition frequency $F_T = 20$ KHz

decimating factor $M = 100$

The filter length $N$ for linear Phase FIR filter is,

$$\boxed{N = \frac{-20 \log_{10}\sqrt{\delta_p \delta_s} - 13}{14.6\, \Delta f}}$$

Normalized transition band width

$$\Delta f = \frac{F_s - F_p}{F_T}$$

$$= \frac{50-40}{20,000}$$

$$\Delta f = \frac{10}{20,000} \Rightarrow \frac{1}{2000}$$

$$\therefore N = \frac{-20 \log_{10}\sqrt{(0.01)(0.002)} - 13}{14.6 \left(\frac{1}{2000}\right)}$$

$$\boxed{N = 4,550}$$