UNIT-3

FREQUENCY RESPONSE OF LINEAR PHASE FIR FILTE (or) SYMMETRIC, ANTISYMMETRIC FIR FILTERS

case (i): symmetrical impulse response, N-odd
case (ii): Symmetrical impulse response, N-even
case (iii): ~~Sym~~ Antisymmetrical Impulse Response, N-odd
case (iv): Antisymmetrical Impulse Response, N-even.

case I: Symmetrical Impulse Response, N-odd

Frequency response of Impulse response for N=7 can be written as $H(e^{j\omega}) = \sum_{n=0}^{6} h(n)e^{-j\omega n}$

- can be splitted as $H(e^{j\omega}) = \sum_{n=0}^{2} h(n)e^{-j\omega n} + h(3)e^{-j3\omega} + \sum_{n=4}^{6} h(n)e^{-j\omega n}$

- In general for N samples

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{N-1}{2}\right)e^{-j\omega(N-1)/2} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n)e^{-j\omega n} \quad \text{—— (1)}$$

Let $n = N-1-m$ $\quad \frac{N+1}{2} = N-1-m$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{N-1}{2}\right)e^{-j\omega(N-1)/2} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m)e^{-j\omega(N-1-m)} \quad \text{—— (2)}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{N-1}{2}\right)e^{-j\omega(N-1)/2} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n)e^{-j\omega(N-1-n)} \quad \text{—— (3)}$$

For symmetrical impulse response

$$h(n) = h(N-1-n)$$

Sub in eqn (3).

$$= e^{-j\omega(N-1)/2}\left[\sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos\omega\left(\frac{N-1}{2}-n\right)+h\left(\frac{N-1}{2}\right)\right]$$

let $\frac{N-1}{2} - n = P$ then

$$H(e^{j\omega}) = e^{-j\omega(N-1)/2}\left[\sum_{P=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-P\right)\cos\omega P+h\left(\frac{N-1}{2}\right)\right]$$

$$= e^{-j\omega(N-1)/2}\left[\sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2}-n\right)\cos\omega n+h\left(\frac{N-1}{2}\right)\right]$$

$$= e^{-j\omega(N-1)/2}\sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n \quad \text{——— (4)}$$

Where $a(0) = h\left(\frac{N-1}{2}\right)$ $\quad a(n) = 2h\left(\frac{N-1}{2}-n\right)$

Eqn (4) can be written as.

$$H(e^{j\omega}) = e^{-j\omega(N-1)/2}\,\bar{H}(e^{j\omega}) = \bar{H}(e^{j\omega})e^{j\theta(\omega)} \quad \text{——— (5)}$$

Where $\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$ ——— (6)

& $\theta(\omega) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega$ ——— (7)

$\bar{H}(e^{j\omega})$ is a real & even fn of ω

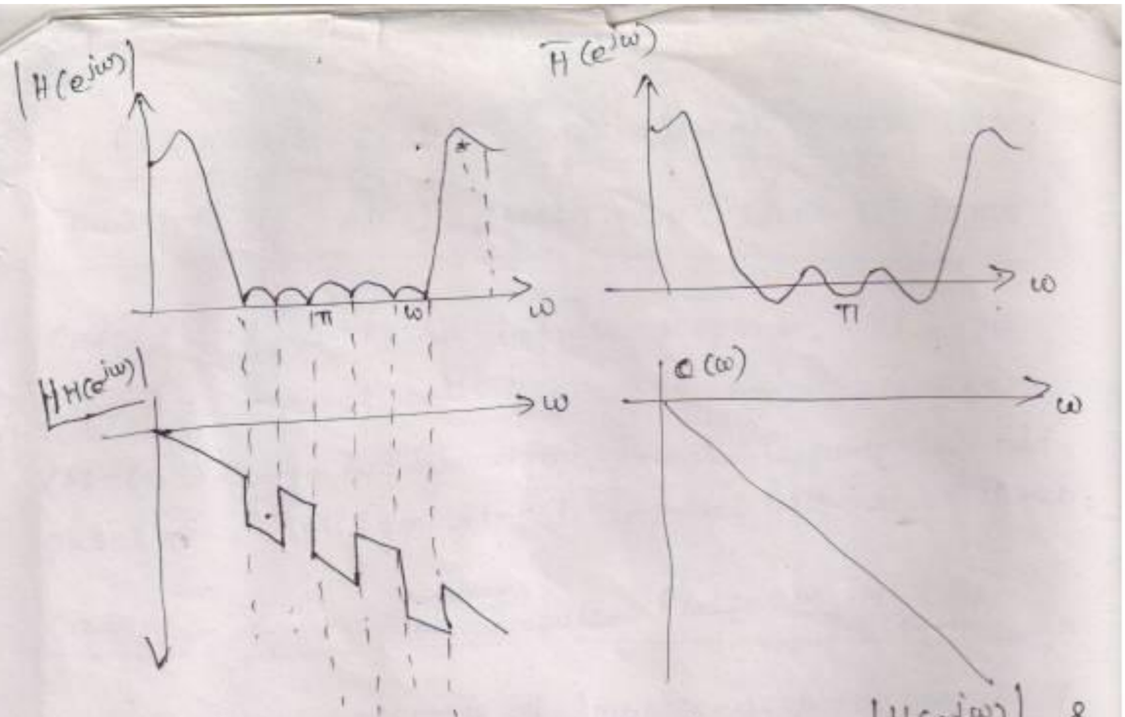
Magnitude $|H(e^{j\omega})| = |\bar{H}(e^{j\omega})|$ &

phase $\angle H(e^{j\omega}) = \theta(\omega) = -\alpha\omega$ when $\bar{H}(e^{j\omega}) \geq 0$

$\angle H(e^{j\omega}) = -\alpha\omega + \pi$ when $\bar{H}(e^{j\omega}) < 0$

and $\theta(\omega) = -\alpha\omega$ when $\bar{H}(e^{j\omega}) < 0$

$\bar{H}(e^{j\omega}) \leftarrow$ zero-phase frequency response.



(fig) Relation b/w Magnitude response, $|H(e^{j\omega})|$ & zero phase response $\bar{H}(e^{j\omega})$ & Relation b/w Phase Response $\angle H(e^{j\omega})$ & $\theta(\omega)$

Case (ii) Symmetric Impulse Response for 'N' even

Fourier Series Method

→ Freq Response $H(e^{j\omega})$ of sys is periodic in 2π.

→ From FS analysis we know that any periodic fn can be expressed as a linear combination of complex exponentials

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

$h_d(n)$ → Fourier Coeffs

→ desired Impulse response of filter

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

Z-TF of sequence $H(z) = \sum_{n=-\infty}^{\infty} h_d(n) z^{-n}$

Eqn rep as non-causal digital filter of ∞ duration.

* To get FIR filter TF, truncate the series

$$h(n) = h_d(n) \quad \text{for } |n| \le \frac{N-1}{2}$$
$$= 0 \quad \text{otherwise}$$

$$H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n) z^{-n}$$

$$= \cancel{h\left(\tfrac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \dots + h(1) z^{-1} +}$$

$$h\left(-\left(\tfrac{N-1}{2}\right)\right) z^{\frac{N-1}{2}} + \dots h(-2) z^{2} + h(-1) z^{1} + h(0) z^{0}$$
$$+ h(1) z^{-1} + h(2) z^{-2} + \dots h\left(\tfrac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)}$$

$$= h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n) z^{-n} + h(-n) z^{n}\right]$$

For Symmetrical impulse response having Symmetry at $n=0$.

$$h(-n) = h(n)$$

$$\cancel{H(z) = h} \quad H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) \left[z^{-n} + z^{n}\right]$$

The TF is not physically realizable. So Multiply by $z^{-\left(\frac{N-1}{2}\right)}$ $\frac{N-1}{2}$ ← delay in samples.

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$
$$= z^{-\left(\frac{N-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left(z^{-n}+z^{n}\right)\right]$$

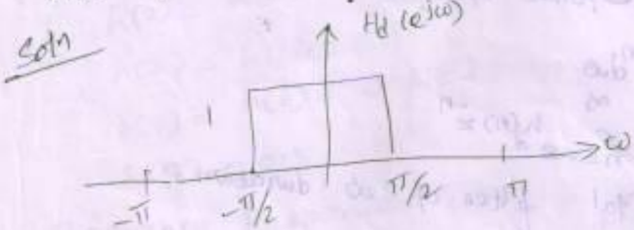
Ex: Design an ideal LPF with freq response

$H_d(e^{j\omega}) = 1$ for $-\pi/2 \le \omega \le \pi/2$

$= 0$ for $\pi/2 \le |\omega| \le \pi$

Find values of $h(n)$ for $N=11$. Plot magnitude Response.

Soln



From freq response, $\alpha = 0$

∴ we get a non causal filter coeffs symmetrical abt $n=0$,

i.e. $h_d(n) = h_d(-n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi j n}\left[e^{jn\pi/2} - e^{-jn\pi/2}\right]$$

$$= \frac{1}{2\pi j n}\, 2j \sin(n\pi/2)$$

$$= \frac{\sin n\pi/2}{n\pi} \quad -\infty \le n \le \infty$$

Truncate $h_d(n)$ to 11 samples,

$$h(n) = \frac{\sin n\pi/2}{n\pi} \quad \text{for } |n| \le 5$$
$$= 0, \text{ otherwise}$$

For $n=0$, $h(0) = \frac{\sin 0}{0} = 0/0$ indeterminate. So.

Apply L'Hospital rule or Substitute n=0 in ∫ formula.

$$h(0) = \lim_{n\to 0} \frac{\sin n\pi/2}{\pi n}$$

x'ly & ÷ by 1/2

$$= \lim_{n\to 0} \frac{1/2 \sin n\pi/2}{\pi n \times 1/2}$$

$$= 1/2 \lim_{n\to 0} \frac{\sin n\pi/2}{n\pi/2} \qquad \left\{ \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 \right.$$

$$= 1/2 \, [1]$$

(or)

Sub n=0 in $h_d(0) = h(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega(0)} d\omega = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1 \, d\omega$

$$= \frac{1}{2\pi} [\omega]_{-\pi/2}^{\pi/2} = \frac{1}{2\pi} [\pi/2 - (-\pi/2)] = \frac{1}{2\pi} \pi$$

$$\boxed{h(0) = 1/2}$$

For n=1, $h(1) = h(-1) = \frac{\sin \pi/2}{1(\pi)} = \frac{1}{\pi} = 0.3183$

For n=2, $h(2) = h(-2) = \frac{\sin 2\pi/2}{2\pi} = 0$

For n=3, $h(3) = h(-3) = \frac{\sin 3\pi/2}{3\pi} = \frac{-1}{3\pi} = -0.106$

For n=4, $h(4) = h(-4) = \frac{\sin 4\pi/2}{4\pi} = 0$

For n=5, $h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi} = 0.06366$

Transfer fn $H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n) \, (z^n + z^{-n})\right]$

$$= h(0) + \sum_{n=1}^{5} h(n) [z^n + z^{-n}]$$

$$= 0.5 + h(1)[z^1 + z^{-1}] + h(2)[z^2 + z^{-2}] + h(3)(z^3 + z^{-3})$$

$$+ h(4)[z^4 + z^{-4}] + h(5)[z^5 + z^{-5}]$$

$$= 0.5 + 0.3183 (z^1 + z^{-1}) + 0 + -0.106(z^3 + z^{-3}) + 0$$

$$+ 0.06366 [z^5 + z^{-5}]$$

$$= 0.5 + 0.3183 (z^1 + z^{-1}) - 0.106 (z^3 + z^{-3}) + 0.06366 (z^5 + z^{-5})$$

TF of realizable filter is $H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$

$$= z^{-5}\left[0.5 + 0.3183\,(z+z^{-1}) - 0.106\,(z^{3}+z^{-3}) + 0.06366\,(z^{5}+z^{-5})\right]$$

$$= 0.06366 + 0.06366 z^{-10} - 0.106 z^{-2} - 0.106 z^{-8} + 0.3183 z^{-4} + 0.3183 z^{-6} + 0.5 z^{-5}$$

$h(0) = h(10) = 0.06366$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

$h(4) = h(6) = 0.3183$

$h(5) = 0.5$

0.5
0.6269
-0.1836
0.0816

Frequency Response $\bar{H}(e^{j\omega}) = \sum_{n=0}^{5} a(n)\cos \omega n$.

$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$

$a(n) = 2h\left(\frac{N-1}{2} - n\right)$

$a(1) = 2h(5-1) = 2h(4) = 2 \times 0.3183 = 0.6366$

$a(2) = 2h(5-2) = 2h(3) = 0$

$a(3) = 2h(5-3) = 2h(2) = 2 \times (-0.106) = -0.212$

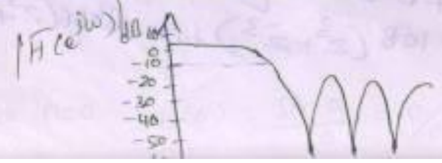
$a(4) = 2h(5-4) = 2h(1) = 0$

$a(5) = 2h(5-5) = 2h(0) = 2 \times 0.06366 = 0.127$

$$\bar{H}(e^{j\omega}) = 0.5 + 0.6366\cos\omega + 0 + (-0.212)\cos 3\omega + 0 + 0.127\cos 5\omega$$

Magnitude in dB $|\bar{H}(e^{j\omega})|_{dB} = 20 \log |\bar{H}(e^{j\omega})|$

ω (in degrees)	0	10	20	30	40	50	60	70	80	90	100	110
$\lvert\bar{H}(e^{j\omega})\rvert_{dB}$	0.4	0.21	-0.26	-0.517	-0.21	0.42	0.77	0.21	-7.79	-6	-14.56	-31.89



H.W
Design ideal HPF with freq resp
$H(e^{j\omega}) = 1$ for $\pi/4 \le |\omega| \le \pi$
$= 0$ for $|\omega| \le \pi/4$
[illegible] plot

LINEAR PHASE FIR FILTERS

The transfer function of a FIR causal filter is

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \quad \text{--- (1)}$$

$h(n) \leftarrow$ Impulse Response of the filter

Fourier transform of $h(n)$ is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} \quad \text{--- (2)}$$

periodic in ~~with~~ frequency with period 2π

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| e^{j\theta(\omega)} \quad \text{--- (3)}$$

Magnitude Response

$\theta(\omega) \leftarrow$ phase Response.

Phase delay of filter $\tau_p = \dfrac{-\theta(\omega)}{\omega}$

Group delay of filter $\tau_g = \dfrac{-d\,\theta(\omega)}{d\omega}$ --- (4)

For FIR filters with linear phase $\theta(\omega) = -\alpha\omega$ --- (5) $\quad -\pi \le \omega \le \pi$

$\alpha \leftarrow$ constant phase delay in sample.

Sub eqn (5) in (4)

$$\tau_p = \frac{-(-\alpha\omega)}{\omega} = \alpha$$

$$\tau_g = -\frac{d}{d\omega}(-\alpha\omega) = +\alpha$$

means α is independent of frequency

* we can write $\displaystyle\sum_{n=0}^{N-1} h(n) e^{-j\omega n} = \pm |H(e^{j\omega})| e^{j\theta(\omega)}$ --- (6)

$$\sum_{n=0}^{N-1} h(n) [\cos \omega n - j \sin \omega n] = \pm |H(e^{j\omega})| [\cos\theta(\omega) + j\sin\theta(\omega)]$$

Taking the ratio of eqn (8) to eqn (7) we get

$$\frac{-\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\sin\alpha\omega}{\cancel{\cos\alpha\omega}} \quad \frac{\pm|H(e^{j\omega})|\sin\theta(\omega)}{\pm|H(e^{j\omega})|\cos\theta(\omega)}$$

$\boxed{\theta(\omega) = -\alpha\omega}$ for linear phase

$$= \cancel{\tan\theta(\omega)} \; \frac{\sin(-\alpha\omega)}{\cos(-\alpha\omega)}$$

$$=$$

$$\frac{\cancel{-}\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\cancel{-}\sin\alpha\omega}{\cos\alpha\omega} \qquad \text{——— (9)}$$

$\sin\omega n \cos\alpha\omega = \sin\alpha\omega\cos\omega n$

$\sin\omega n\cos\alpha\omega - \cos\omega n\sin\alpha\omega = 0$

$\{\sin A\cos B - \cos A\sin B$

$\sin$

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha - n)\omega = 0 \qquad \text{——— (10)}$$

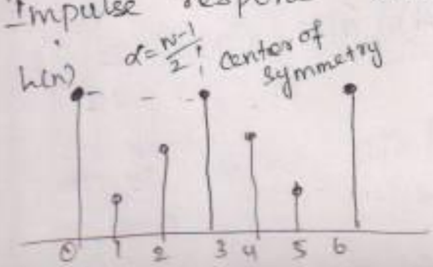
Eqn (10) will become zero when $h(n) = h(N-1-n)$ ——— (11)

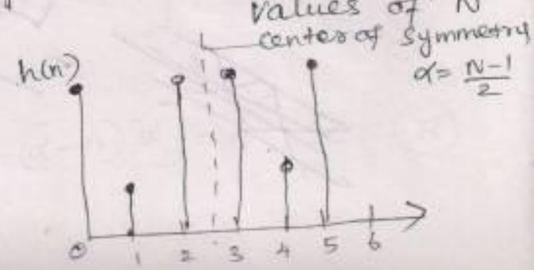
and $\alpha = \frac{N-1}{2}$ ——— (12)

There fore FIR filters will have constant phase delay, constant group delay

* constant phase delay, constant group delay when impulse response is symmetrical about $\alpha = \frac{N-1}{2}$

Impulse respone satisfying (11) & (12) for odd & even values of 'N'





If only constant group delay (τ_g) is required, not the phase delay we can write

$$\theta(\omega) = \beta - \alpha\omega \quad \text{--- (13)}$$

w.k.t

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| e^{j\theta(\omega)}$$

$$= \pm |H(e^{j\omega})| e^{j(\beta - \alpha\omega)} \quad \text{--- (14)}$$

$$\sum_{n=0}^{N-1} h(n) e^{-j\omega n} = \pm |H(e^{j\omega})| e^{j(\beta - \alpha\omega)} \quad \text{--- (15)}$$

$$\sum_{n=0}^{N-1} h(n)[\cos\omega n - j\sin\omega n] = \pm |H(e^{j\omega})| [\cos(\beta-\alpha\omega) + j\sin(\beta-\alpha\omega)]$$

Equating Real & Imaginary parts

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm |H(e^{j\omega})| \cos(\beta - \alpha\omega) \quad \text{--- (16)}$$

$$-\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm |H(e^{j\omega})| \sin(\beta - \alpha\omega) \quad \text{--- (17)}$$

Take ratio of eqn (17) ~~by~~ to eqn (16)

$$\frac{-\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)}$$

$$\sum_{n=0}^{N-1} h(n)\sin[(\beta - \alpha\omega) - \omega n] = 0$$

$$\sum_{n=0}^{N-1} h(n)\sin[\beta - (\alpha - n)\omega] = 0 \quad \text{--- (18)}$$

If $\beta = \pi/2$; sub β in (18)

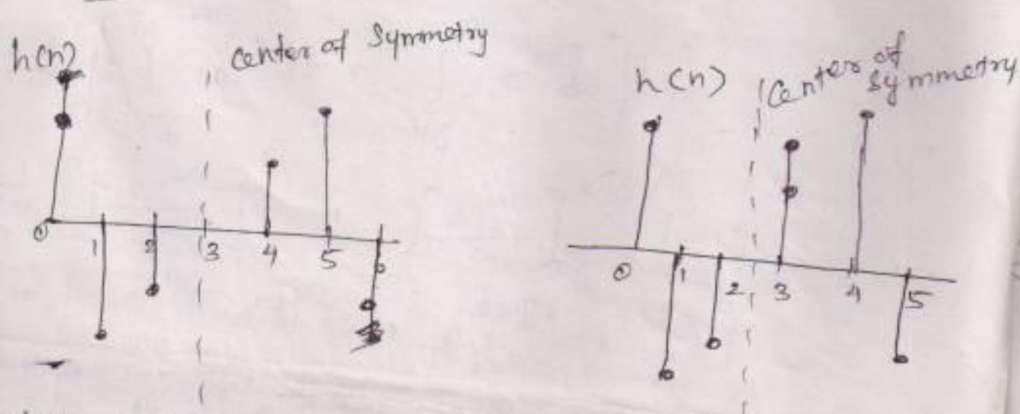
Eqn (19) will be satisfied when

$$h(n) = -h(N-1-n) \quad \text{———— (20)}$$

&

$$\alpha = \frac{N-1}{2} \quad \text{———— (21)}$$

∴ FIR filters have constant group delay τ_g, and not constant phase delay τ_p when the impulse response is antisymmetrical about $\alpha = \frac{N-1}{2}$.



Impulse response sequences satisfying

$$h(n) = -h(N-1-n) \; \& \; \alpha = \frac{N-1}{2}$$

When N = 7 center of symmetry occurs in 3rd sample

When N = 6, center of symmetry occurs at 2½ sample

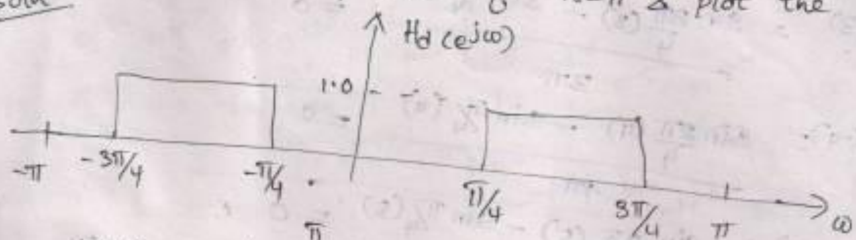
$h\left(\frac{N-1}{2}\right) = 0$ for antisymmetric odd sequence.

Design an ideal BPF with a freq response.

$$H_d(e^{j\omega}) = 1 \quad \text{for } \pi/4 \le |\omega| \le 3\pi/4$$
$$= 0 \quad \text{otherwise.}$$

Find the values of $h(n)$ for $N=11$ & plot the freq response.

Soln



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} 1\,e^{j\omega n}\,d\omega + \int_{\pi/4}^{3\pi/4} 1\,e^{j\omega n}\,d\omega\right]$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{j\omega n}}{jn}\right]_{-3\pi/4}^{-\pi/4} + \left[\frac{e^{j\omega n}}{jn}\right]_{\pi/4}^{3\pi/4}\right\}$$

$$= \frac{1}{2\pi jn}\left[e^{-j\pi/4\,n} - e^{-j3\pi/4\,n} + e^{j3\pi/4\,n} - e^{+j\pi/4\,n}\right]$$

$$= \frac{1}{2\pi jn}\left\{e^{j3\pi/4\,n} - e^{-j3\pi/4\,n} - \left[e^{j\pi/4} - e^{-j\pi/4}\right]\right\}$$

$$= \frac{1}{2\pi jn}\left\{2j\sin\frac{3\pi}{4}n - 2j\sin\frac{\pi}{4}n\right\}$$

$$= \frac{1}{n\pi}\left[\sin\frac{3\pi}{4}n - \sin\frac{\pi}{4}n\right] \quad -\infty \le n \le \infty$$

...cate $h_d(n)$ to 11 samples.

$$h(n) = h_d(n) \quad \text{for } |n| \le 5$$
$$= 0 \quad \text{otherwise.}$$

coeffs are symmetrical about $n=0$, satisfies the $h(n) = h(-n)$

$$h(0) = \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} d\omega + \int_{\pi/4}^{3\pi/4} d\omega\right]$$

$$= \frac{1}{2\pi}\left\{[\omega]^{-\pi/4} \ldots\right.$$

$n=1,\quad h(1)=h(-1)=\dfrac{\sin 3\pi/4-\sin \pi/4}{\pi}=0$

$n=2,\quad h(2)=h(-2)=\dfrac{\sin \frac{3\pi}{4}(2)-\sin \frac{\pi}{4}(2)}{2\pi}=\dfrac{-2}{2\pi}=-0.3183$

$n=3,\quad h(3)=h(-3)=\dfrac{\sin \frac{3\pi}{4}(3)-\sin \frac{\pi}{4}(3)}{3\pi}=0$

$n=4,\quad h(4)=h(-4)=\dfrac{\sin \frac{3\pi}{4}(4)-\sin \frac{\pi}{4}(4)}{4\pi}=0$

$n=5,\quad h(5)=h(-5)=\dfrac{\sin \frac{3\pi}{4}(5)-\sin \frac{\pi}{4}(5)}{5\pi}=0$

Transfer fn of filter.

$$H(z)=h(0)+\sum_{n=1}^{\frac{N-1}{2}}\left[h(n)(z^{-n}+z^{n})\right]$$

$$=0.5+\left[h(1)(z^{-1}+z^{1})+h(2)(z^{-2}+z^{2})+h(3)(z^{-3}+z^{3})+h(4)(z^{-4}+z^{4})+h(5)(z^{5}+z^{-5})\right]$$

$$=0.5+0+(-0.3183)(z^{-2}+z^{2})+0$$

$$=0.5-0.3183(z^{-2}+z^{2})$$

Transfer fn of physically realizable filter

$$H'(z)=z^{-\frac{(N-1)}{2}}H(z)$$

$$=z^{-5}H(z)$$

$$=z^{-5}\left[0.5-0.3183(z^{-2}+z^{2})\right]$$

$$=0.5z^{-5}-0.3183z^{-7}-0.3183z^{-3}$$

Filter coeffs.

$h(0)=h(10)=$

$h(1)=h(9)=0$

$h(2)=h(8)=0$

$h(3)=h(7)=-0.3183$

$h(4)=h(6)=0$

$h(5)=0.5$

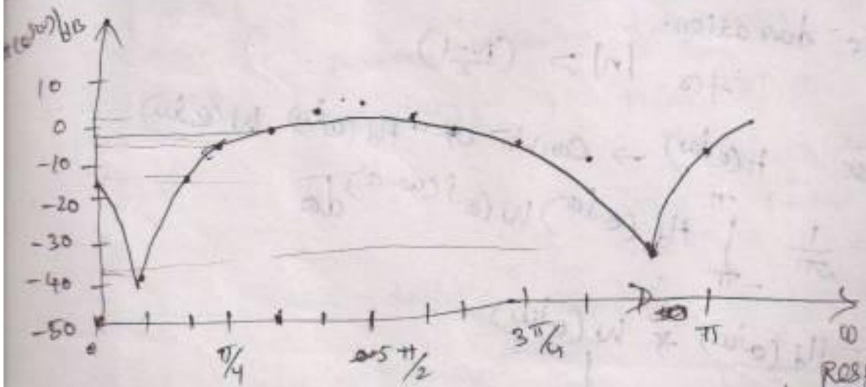
$$H(e^{j\omega})=\sum_{n=1}^{\frac{N-1}{2}}a(n)\cos\omega n$$

$a(0)=h\left(\frac{N-1}{2}\right)=h(5)=0.5$

$$\bar{H}(e^{j\omega}) = 0.5 - 0.6366 \cos 2\omega$$

$$|\bar{H}(e^{j\omega})|_{dB} = 20 \log |\bar{H}(e^{j\omega})|$$

ω (degree)	0	20	30	45	60	75	90	105	120	135	150	160	180
$\bar{H}(e^{j\omega})$	-0.1366	0.012	0.1817	0.5	0.818	1.05	1.1366	1.05	0.818	0.5	0.1817	0.012	-0.1366
$\bar{H}(e^{j\omega})$ dB	-17.3	-38.17	-14.8	-6.02	-1.74	0.4346	1.11	0.4846	-1.74	-6.02	-14.8	-38.17	-17.3



Response of BPF.

A.W Design an ideal BRF with a desired freq response.

$H_d(e^{j\omega}) = 1$ for $|\omega| \le \pi/3$ & $|\omega| \ge 2\pi/3$

$= 0$ otherwise.

Design of FIR filters using windows

$H_d(e^{j\omega}) \rightarrow$ Desired freq response of a filter is periodic in frequency.

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega.$$

$\rightarrow$ Fourier coeffs having ∞ length.

* Truncate ∞ fourier series at $n = \pm \left(\frac{N-1}{2}\right)$

To reduce these oscillations, Fourier coeffs of filter are modified by xlying ∞ imp response $\bar{c}$ a finite weighing sequence $w(n)$ called window

$$\omega(n) = w(-n) \neq 0 \quad \text{for } |n| \leq \left(\frac{N-1}{2}\right)$$
$$= 0 \quad \text{for } |n| > \left(\frac{N-1}{2}\right).$$

$$h(n) = h_d(n)\,w(n) \quad \text{for all } |n| \leq \left(\frac{N-1}{2}\right)$$
→ finite duration.
$$= 0 \quad \text{for } |n| > \left(\frac{N-1}{2}\right)$$

Frequency response $H(e^{j\omega}) \rightarrow$ Convln of $H_d(e^{j\omega}) \cdot W(e^{j\omega})$

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta})\, W(e^{j(\omega-\theta)})\, d\theta$$

$$= \underbrace{H_d(e^{j\omega})}_{\text{Periodic}} * \underbrace{W(e^{j\omega})}_{\text{Periodic}}$$

Periodic Convln.

FT of window consists of Central lobe, side lobes.

Central lobe contains most of energy of window.

$H(e^{j\omega})$ dep of freq resp of window $(W(e^{j\omega}))$

∴ window chosen for truncating ∞ imp response should have some desirable charac

1) Central lobe of freq resp of window should contain most of the energy & shd be narrow
2) Highest side lobe level of freq resp shd be small
3) SLs of freq resp should ↓ in energy rapidly as $\omega \rightarrow 0$