UNIT-I DISCRETE FOURIER TRANSFORM AND COMPUTATION

1. What is DFT?

It is a finite duration discrete frequency sequence, which is obtained by sampling one period of Fourier transform. Sampling is done at N equally spaced points over the period extending from w=0 to 2л.

2. Define N point DFT.

The DFT of discrete sequence x(n) is denoted by X(K). It is given by,

Here k=0,1,2…N-1

Since this summation is taken for N points, it is called as N-point DFT.

3. What is DFT of unit impulse δ(n)?

The DFT of unit impulse δ(n) is unity.

4. List the properties of DFT.

- Linearity
- Periodicity
- Circular symmetry
- Symmetry
- Time shift
- Frequency shift
- complex

5. State Linearity property of DFT.

DFT of linear combination of two or more signals is equal to the sum of linear co-efficient of DFT of individual signal.

6. When a sequence is called circularly even?

The N point discrete time sequence is circularly even if it is symmetric about point on the circle.

7. What is the condition of a sequence to be circularly odd?

An N point sequence is called circularly odd it if is anti symmetric about point zero on the circle.

8. Why the result of circular and linear convolution is not same?

Circular convolution contains same number of samples as that of x (n) and h(n), while in linear convolution, number of samples in the result (N) are, N=L+M-1

Where L=Number of samples in x (n)

M=Number of samples in h (n)

9. What is circular time shift of sequence?

Shifting the sequence in time domain by '1' samples is equivalent to multiplying the sequence in frequency domain by W_N^{kl}

10. What is the disadvantage of direct computation of DFT?

For the computation of N-point DFT, N2 complex multiplications and N[N-1] Complex additions are required. If the value of N is large than the number of into lakhs. This proves inefficiency of direct DFT computation.

11. What is the way to reduce number of arithmetic operations during Decomputation?

Number of arithmetic operations involved in the computation of DFT is greatly reducing different FFT algorithms as follows.

1. Radix-2 FFT algorithms.
   - Radix-2 Decimation in Time (DIT) algorithm.
   - Radix-2 Decimation in Frequency (DIF) algorithm.
2. Radix-4 FFT algorithm.

12. What is the computational complexity using FFT algorithm?

1. Complex multiplications = $N/2 \log^2 N$
2. Complex additions = $N \log^2 N$

13. How linear filtering is done sing FFT?

Thus, by folding the sequence h (n), e(n) can compute the linear filtering using FFT.

Correlation is the basic process of doing linear filtering using FFT. The correlation is the sequence x(n) has a length L. If we want to find the N point DFT (N>L) of the sequence x (n).

14. What is zero padding? What are its uses?

This is known as zero padding. The uses of padding a sequence with zeros are

(i) We can get 'better display' of the frequency spectrum.

(ii) With zero padding, the DFT can be used in linear filtering.

15. Why FFT is needed?

The direct evaluation of the DFT using the formula requires N^2 complex multiplications and N (N-1) complex additions. Thus for reasonably large values of N (in order of 1000) direct evaluation of the DFT requires an inordinate amount of computation. By using FFT algorithms the number of computations can be reduced.

For example, for an N-point DFT,the number of complex multiplications required using FFT is $N/2\log_2 N$. If N=16, the number of complex multiplications required for direct evaluation of DFT is 256, whereas using DFT only 32 multiplications are required.

16. What is the speed of improvement factor in calculating 64-point DFT of a sequence using direct computation and computation and FFT algorithms?

(Or) Calculate the number of multiplications needed in the calculation of DFT and FFT with 64-point sequence.

The number of complex multiplications required using direct computation is N^2 =642=4096.

The number of complex multiplications required using FFT is $N/2 \log_2 N = 64/2 \log_2 64 = 192$.

Speed improvement factor = 4096/192=21.33

17. What is the main advantage of FFT?

FFT reduces the computation time required to compute discrete Fourier transform.

18. How many multiplications and additions are required to compute N-point DFT using redix-2 FFT?

The number of multiplications and additions required to compute N-point DFT using redix-2 FFT are $N \log_2 N$ and $N/2 \log_2 N$ respectively.

19. What is meant by radix-2 FFT?

The FFT algorithm is most efficient in calculating N-point DFT. If the number of output points N can be expressed as a power of 2, that is, $N=2^M$, where M is an integer, Then this algorithm is known as radix-2 FFT algorithm.

20. What are the applications of FFT algorithms?

- Linear filtering
- Correlation
- Spectrum analysis

21. What is a decimation-in-frequency algorithm?

In this the output sequence X (K) is divided into two N/2 point sequences and ch each N/2 point sequences are in turn divided into two N/4 point sequences.

22. Distinguish between DFT and DTFT.

s.no	DFT	DTFT
1.	Obtained by performing sampling operation in both time and frequency domains.	Sampling is performed only in time domain
2.	Discrete frequency spectrum	Continuous function of ω

1. (i) Convolution property of DFT (6)

If $DFT[x_1(n)] = X_1(k)$ and $DFT[x_2(n)] = X_2(k)$

then $\boxed{x_1(n) \; (N) \; x_2(n) \xleftrightarrow{DFT} X_1(k) X_2(k)}$

(ii) Find the inverse DFT of (10)

$X(K) = \{7, -\sqrt{2}-j\sqrt{2}, -j, \sqrt{2}-j\sqrt{2}, 1, \sqrt{2}+j\sqrt{2}, j, -\sqrt{2}+j\sqrt{2}\}$

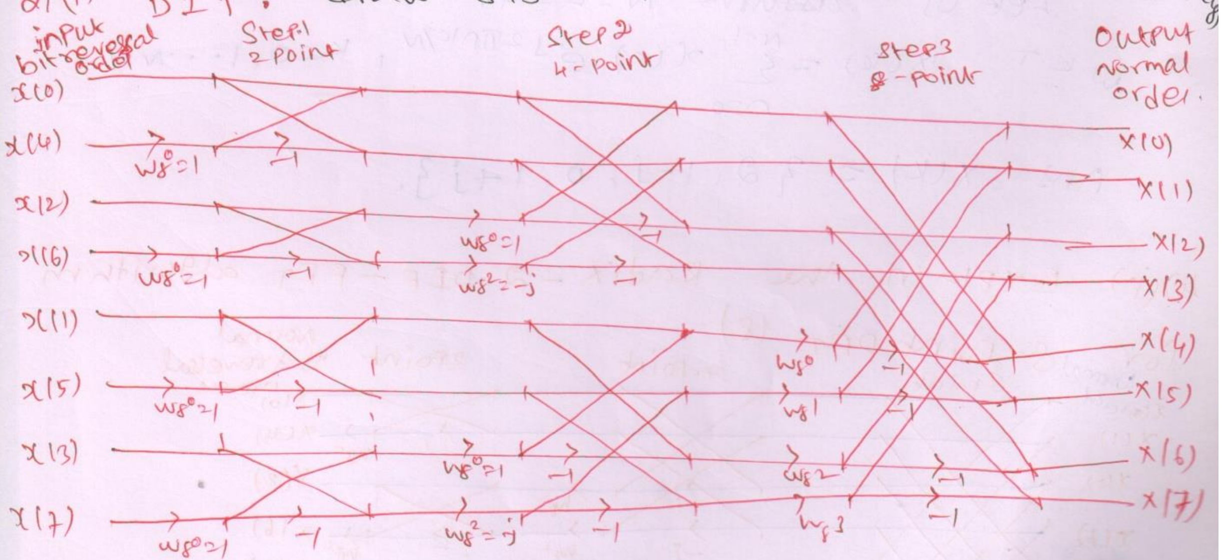
IDFT formulae, $x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j\frac{2\pi nk}{N}}, n = 0, 1 \cdots (N-1)$

Then apply the N = 8 point sequence.

$$\therefore x(n) = \frac{1}{8} \sum_{K=0}^{7} X(K) \; e^{j\frac{n\pi K}{4}}, n = 0, 1, \cdots 7.$$

$x(n) =$

2) (i) Derive the decimation-in time radix-2 FFT algorithm and draw signal flow graph for 8-point sequence DIT.



2)(ii) $x(n) = \{2,2,2,2,1,1,1,1\}$. (8) Using FFT algorithm, Compute the DFT of the sequence

By using FFT-DIT algorithm	FFT-DIF algorithm
1. Input is bit reversal order	Input is normal order
2. Output is Normal order	Output is bit reversal order
3. 2-point, 4-point & 8-point sequences	8-point, 4-point and 2-point sequences.

3)(i) Explain the following properties of DFT (10)

a) Time shifting.

If $DFT[x(n)] = X(k)$

$DFT[x((-n))_N] = DFT[x(N-n)]$

$= X((-K))_N = X(N-K)$.

b) Conjugate Symmetry.

If $DFT[x(n)] = X(k)$

then $DFT[x^*(n)] = X^*(N-K)$

$= X^*((-k))_N$.

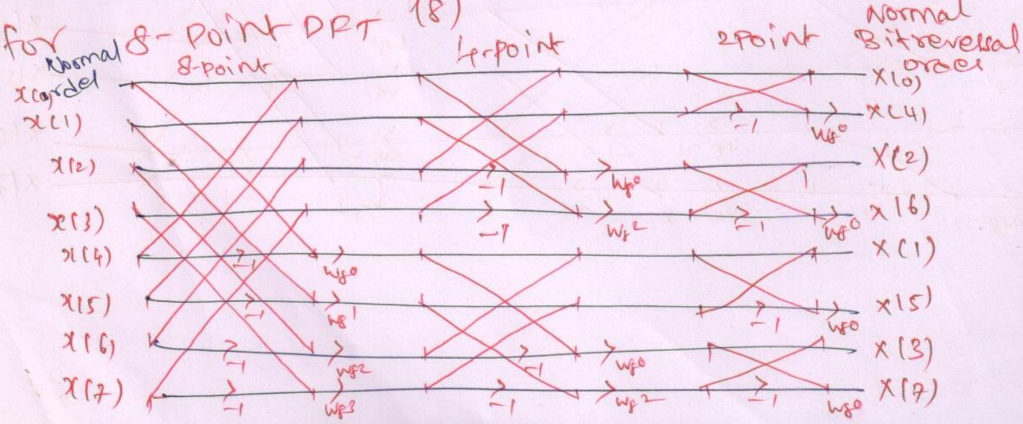
3)(ii) Compute the 4-point DFT of $x(n) = \{1, 0, 0, 0\}$. (6)

Let us assume $N = L = 4$.

W.k.T $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$, $k = 0, 1 \ldots N-1$

Ans. $X(k) = \{2, 1-j, 0, 1+j\}$.

4)(i) Explain the Radix-2 DIF-FFT algorithm for 8-point DFT (8)



3.3

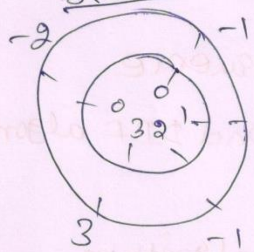
4)ii) Obtain the 8-point DFT using DIT-FFT algorithm. $x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$.

Answer

$X(k) = \{6, -0.707 - j1.707, 1 - j, 0.707 + j0.293, 0, 0.707 - j0.293, 1 + j, -0.707 + j1.707\}$.

5) Find the circular convolution of two finite duration sequences $x_1(n) = \{1, -1, -2, 3, -1\}$ and $x_2(n) = \{1, 2, 3\}$.

Step 1



$y(0) = 1(1) + 0(-1) + 0(-2) + 3(3) + 2(-1)$

$= 8$

Step 2

$y(1) = 1(2) + (-1)1 + (-2)0 + 3(0) + 3(-1)$

$\boxed{y(1) = -2}$

$y(2) = -1$

$y(3) = -4$

$y(4) = -1$

Ans: $\boxed{y(n) = \{8, -2, -1, -4, -1\}}$

6) Compute the 8-point DFT of the sequence $x(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}$ by using DIF-FFT algorithms

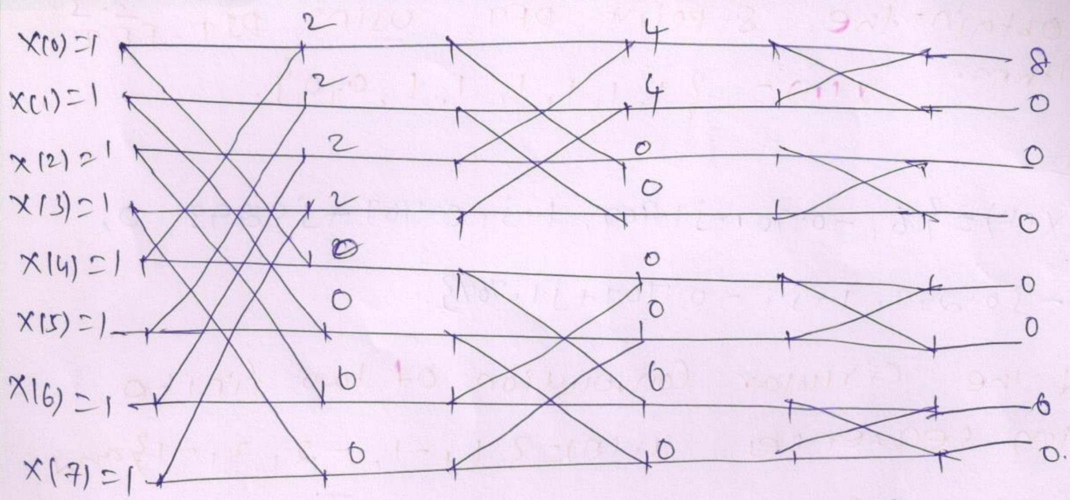
The twiddle factors associated with butterflie can be found as.

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$



Ans. $X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$

(10) Compute 4-point DFT of a sequence $x(n) = \{0, 1, 2, 3\}$ using DIT and DIF algorithm

DIT algorithm

Input	S_1	Output
0	$0+2=2$	$2+4=6$
2	$0-2=-2$	$-2+(-j)(-2) = -2+2j$
1	$1+3=4$	$2-4=-2$
3	$1-3=-2$	$-2-(-j)(-2) = -2-2j$

$X(k) = \{6, -2+2j, -2, -2-2j\}$

DIF algorithm

Input	S_1	Output
0	$0+2=2$	$2+4=6$
1	$1+3=4$	$2-4=-2$
2	$0-2=-2$	$-2+2j$
3	$(1-3)(-j)=2j$	$-2-2j$

$X(k) = \{6, -2+2j, -2, -2-2j\}$

PART A

UNIT II IIR FILTERS

1. What is a digital filter?

A digital filter is a device that eliminates noise and extracts the signal of interest from other signals.

2. Analog filters are composed of which parameters?

1). pass band 2).stop band 3).Cut-off frequency

3. Define pass band.

It passes certain range of frequencies. In this, attenuation is zero.

4. Define stop band.

It suppresses certain range of frequencies. In this, attenuation is infinity.

5. What is mean by cut-off frequency?

This is the frequency which separates pass band and stop band.

6. What is the difference between analog and digital filters?

Analog filters are designed using analog components (R,L,C) while digital filters are implemented using difference equation and implemented using software.

7. What are the basic types of analog filters?

1). Low pass filter – LPF
2).High pass filter – HPF
3).Band pass filter - BPF
4).Band stop filter - BSF

8. What is the condition for digital filter to be realized?

The impulse response of filter should be causal, $h(n) = 0$ for $n<0$.

9. Why ideal frequency selective filters are not realizable?

Ideal frequency selective filters are not realizable because they are non- causal. That is, its impulse response is present for negative values of "n" also.

10. For IIR filter realization what is required?

Present, past, future samples of input and past values of output are required.

11. Why IIR systems are called recursive systems?

Because the feedback connection is present from output side to input

12. Which types of structures are used to realize IIR systems?

1). Direct form structure
2).Cascade form structure
3).Parallel form structure

13. Why direct form-II structure is preferred most and why?

The numbers of delay elements are reduced in direct form-II structure compared to direct form-I structure. That means the memory locations are reduced in direct form-II structure.

14. Why direct form-I and direct form-II are called as direct form structures?

The direct form-I and direct form-II structures are obtained directly from the corresponding transfer function without any rearrangements. So these structures are called as direct form structures.

15. What is advantage of direct form structure?

Implementation of direct form is very easy.

16. Give the disadvantage of direct form structure?

Both direct form structures are sensitive to the effects of quantization errors in the coefficients. So practically not preferred

17. What is the use of transpose operation?

If two digital structures have the same transfer function then they are called as equivalent structures. By using the transpose operation, we can obtain equivalent structure from a given realization structure.

18. What is transposition or flow graph reversal theorem?

If we reverse the directions of all branch transmittances and interchange input and output in the flow graph then the system transfer function remains unchanged.

19. How a transposed structure is obtained?

1). Reverse all signal flow graph directions.
2). Change branching nodes into adders and vice-versa.
3). Interchange input and output.

20. Why feedback is required in IIR systems?

It is required to generate infinitely long impulse response in IIR systems.

21. Write the expression for order of Butterworth filter?

The expression is $N=\log(\lambda/\epsilon)^{1/2}/\log(1/k)^{1/2}$

22. Write the expression for the order of chebyshev filter?

$N=\cosh^{-1}(\lambda/e)/\cosh^{-1}(1/k)$

23. Write the various frequency transformations in analog domain?

LPF to LPF: $s=s/\Omega c$
LPF to HPF: $s=\Omega c/s$
LPF to BPF: $s=s2xlxu/s(xu-xl)$
LPF to BSF: $s=s(xu-xl)?s2=xlxu$. $X=\Omega$

24. Write the steps in designing chebyshev filter?

1. Find the order of the filter.
2. Find the value of major and minor axis.
3. Calculate the poles.
4. Find the denominator function using the above poles.
5. The numerator polynomial value depends on the value of n.

If n is odd: put s=0 in the denominator polynomial. If n is even put s=0 and divide it by (1+e2)1/2

25. Write down the steps for designing a Butterworth filter?

1. From the given specifications find the order of the filter
2 find the transfer function from the value of N
3. Find Ωc
4 find the transfer function ha(s) for the above value of Ωc by su s by that value.

26. State the equation for finding the poles in chebyshev filter.

$sk=a\cos\phi k+jb\sin\phi k$, where $\phi k=\pi/2+(2k-1)/2n)\pi$

27. State the steps to design digital IIR filter using bilinear method.

Substitute s by 2/T (z-1/z+1), where $T=2/\Omega$ (tan (w/2) in h(s) to get h (z)

28. What is warping effect or frequency warping? (NOV/DEC-10)

For smaller values of w there exist linear relationship between w and but for Larger values of w the relationship is nonlinear. This introduces distortion in the Frequency axis. This effect compresses the magnitude and phase response. This Effect is called warping effect.

29. Write a note on pre warping or pre scaling. (APRIL/MAY 2011) (MAY/JUNE-2014)

The effect of the non linear compression at high frequencies can be compensated. When the desired magnitude response is piecewise constant over frequency, this Compression can be compensated by introducing a suitable rescaling or pre warping the critical frequencies.

30. Give the bilinear transform equation between s plane and z plane.

s=2/T (z-1/z+1)

31. Give hamming window function.(MAY/JUNE-14)

The equation for Hamming window is given by

WH(n)= 0.54-0.46 cos (2πn/N-1) for $0 \le n \le N-1$

0 otherwise

32. What is meant by impulse invariant method?

In this method of digitizing an analog filter, the impulse response of the resulting digital filter is a sampled version of the impulse response of the analog filter. For e.g. if the transfer function is of the form, 1/s-p, then

H (z) =1/1-e-pTz-1

33. What do you understand by backward difference?

One of the simplest methods of converting analog to digital filter is to approximate the differential equation by an equivalent difference equation.

d/dt(y(t)/t=nT=(y(nT)-y(nT-T))/T

34. What are the significance of chebyshev filter? (NOV/DEC-10)

1. The magnitude response of the chebyshev filter exhibits ripple either in the stop band or the pass band.
2. The poles of this filter lies on the ellipse.

35. Give the Butterworth filter transfer function and its magnitude characteristics for Different orders of filter.

The transfer function of the Butterworth filter is given by

H (jΩ) =1/1+j (Ω/Ωc) N

36. Give the equation for the order N, major, minor axis of an ellipse in case of chebyshev filter?

The order is given by $N=\cosh^{-1}(((10^{.1\alpha p})-1/10^{.1\alpha s}-1)1/2))/\cosh^{-1}\Omega s/\Omega p$

$A= (\mu^{1/N}-\mu^{-1/N})/2\Omega p$

$B=\Omega p\ (\mu^{1/N}+ \mu^{-1/N})/2$

37. How can you design a digital filter from analog filter?

Digital filter can de designed from analog filter using the following methods

1. Approximation of derivatives
2. Impulse invariant method (IIM)
3. Bilinear transformation (BLT)

38. What is a disadvantage of BLT method?

The mapping is non-linear and because of this, frequency warping effect takes place.

39. List the Butterworth polynomial for various orders.

N Denominator polynomial

1).S+1

2).S2+.707s+1

3). (s+1)(s2+s+1)

4). (s2+.7653s+1)(s2+1.84s+1)

5). (s+1)(s2+.6183s+1)(s2+1.618s+1)

6). (s2+1.93s+1)(s2+.707s+1)(s2+.5s+1)

7). (s+1)(s2+1.809s+1)(s2+1.24s+1)(s2+.48s+1)

40. Differentiate Butterworth and Chebyshev filter.

Butterworth damping factor 1.44 and chebyshev is 1.06.Butterworth is flat response .but chebyshev is damped response.

41. What is filter?

Filter is frequency selective devices, which amplify particular range of frequencies and attenuate particular range of frequencies.

42. What are the types of digital filter according to their impulse response?

IIR (Infinite impulse response) filter
FIR (Finite Impulse Response) filter.

43. How phase distortion and delay distortion are introduced?

1. The phase distortion is introduced when the phase characteristics of a filter is Nonlinear within the desired frequency band

2. The delay distortion is introduced when the delay is not constant within the desired frequency band

44. Define IIR filter.

The filters designed by considering all the infinite samples of impulse response are called IIR filter.

45. What is the limitation of approximation of derivative method?

It is suitable only for designing of low pass and band pass IIR digital filters with relatively small resonant frequencies.

PART – B

1. i) Explain the procedure for designing analog filters using the Chebyshev approximation.

   ii) Convert the following analog transfer function in to digital using impulse invariant mapping with T=1sec

$$H(s)=\frac{3}{(s+3)(s+5)}$$

2. i) Design a digital second order low pass Butterworth filter with cut off frequency 2200 Hz using bilinear transformation. Sampling rate is 8000 Hz.

   ii) Determine the cascade form and parallel form implementation of the system governed by the transfer function

$$H(Z)=\frac{(1+Z^{-1}}{(1+2Z^{-1}}$$

3. Design a digital Butterworth filter using impulse invariance method satisfying the constraints .Assume T=1sec

$$0.8\leq \ |H(e^{jw})|\ )\leq 1 \quad : 0\leq w \leq 0.2\pi$$

$$|H(e^{jw})|\ )\leq 0.2;\ 0.6\pi \leq w \leq \pi$$

4. Obtain the direct form I ,direct form II and cascade form realization of the following system functions

$$y(n)=0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2)$$

5. Determine the system function H(z) of the chebyshevs low pass digital filter with the specifications

   1. α_p =1dB ripple in the pass band $0\leq w \leq 0.2\pi$
   2. α_s=1dB ripple in the stop band $0.3\pi \leq w \leq \pi$

   Using bilinear transformation (assume T=1sec)

6. Obtain the direct form I, direct form ii ,cascade, parallel form realization for the system

$$y(n)= -0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2)$$

7. Explain in detail Butterworth filter approximation

8. Explain the bilinear transform method of IIR filter design. What is wrapping effect? Explain the poles and zeros mapping procedure clearly.

9. i)Obtain the cascade form realization of the digital system

$$y(n)=3/4\, y(n-1)-(1/8)y(n-2)+1/3x(n-1)+x(n)$$

ii) Convert the given analog filter with transfer function $H(s)=\frac{2}{(s+1)(s+2)}$ in to a digital IIR filter using bilinear transformation. Assume T=1sec.

10. Discuss the steps in the design of IIR filter using bilinear transformation for any one type of filter?

11. Apply Bilinear Transformation to H(s) =2/(S+2) (S+3) with T=0.1 sec.

12. Design a analog Butterworth filter that has a 2db pass band attenuation at a frequency of 20 r/sec & at least 10db stop band attenuation at 30 r/sec?

ANSWER KEY

1) procedure for designing analog filters using the Chebyshev approximation:

1. From the given specifications find the order of the filter $\bar{N}$
2. Round off it to the next higher integer
3. using the following formulas find the values of a and b, which are minor and major axis of the ellipse respectively

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2}, \quad b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}, \quad \varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

4. Calculate the poles of Chebyshev filter which lies on an ellipse by using the formula.

$$S_k = a\cos\phi_k + jb\sin\phi_k \cdot k = 1, 2 \cdots N$$

$$\text{where, } \phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi, \quad k = 1, 2 \cdots N.$$

2) Given

$$H(s) = \frac{3}{(s+3)(s+5)}$$

by using IIT method.

Solution

using impulse invariance method

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k} \quad \text{then } H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}.$$

3) second order lowpass filter by using bilinear transformation.

Solution

For second order Lowpass filter

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

For biliner transfer function,

Substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to get $H(z)$

$$\therefore H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

4) $0.8 \le |H(e^{j\omega})| \le 1 \; ; \; 0 \le \omega \le 0.2\pi$

$|H(e^{j\omega})| \le 0.2 \; ; \; 0.6\pi \le \omega \le \pi$

Solution

From the data we find $\Omega p = 0.2\pi$, $\Omega s = 0.6\pi$

$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8$ and $\frac{1}{\sqrt{1+\lambda^2}} = 0.2$

$$N \ge \frac{\log\left(\frac{\lambda}{\varepsilon}\right)}{\log\left(\frac{\Omega s}{\Omega p}\right)} = 4$$

For $N=4$, the transfer function of normalised Butterworth filter is

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)}$$

$$\Omega c = \frac{\Omega p}{(10^{0.1\alpha p}-1)^{1/2N}} = \frac{\Omega p}{\varepsilon^{1/N}}$$

5) Butterworth filter approximation:

1. From the given specification find out the order of the filter N

2. Round off it to the next higher integer

3. Find the transfer function $H(s)$ for $\Omega_c = 1$ rad/sec for the value of N

4. Calculate the value of cutoff frequency Ω_c.

5. Find the transfer function $H_a(s)$ for the above value of Ω_c by substituting $s \rightarrow \frac{s}{\Omega_c}$ in $H(s)$

6) Bilinear transformation methods:

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2} \quad \text{and} \quad \omega = 2\tan^{-1}\frac{\Omega T}{2}$$

The warping effect

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

For small values of ω, $\Omega = \frac{\cancel{2}}{T}\cdot\frac{\omega}{\cancel{2}} = \frac{\omega}{T}$

$$\boxed{\omega = \Omega T}$$

Prewarping:

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$$

Substitude $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ into the transfer function found in the analog frequencies find $H(s)$ of the analog filter

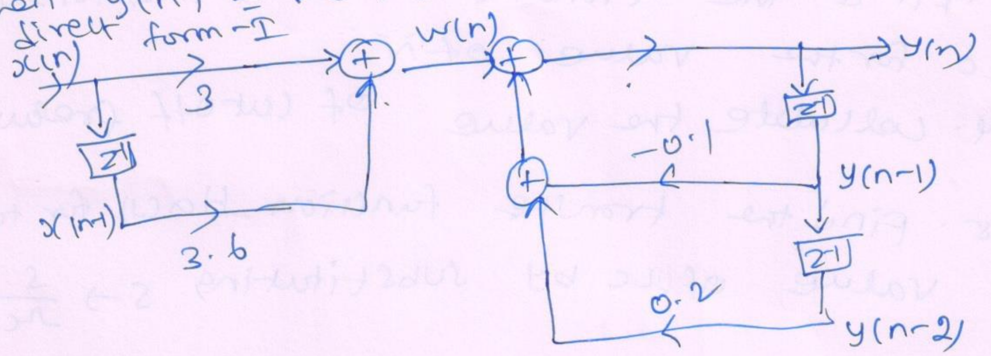
7) $y(n) = -0.1\, y(n-1) + 0.2\, y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

Solution

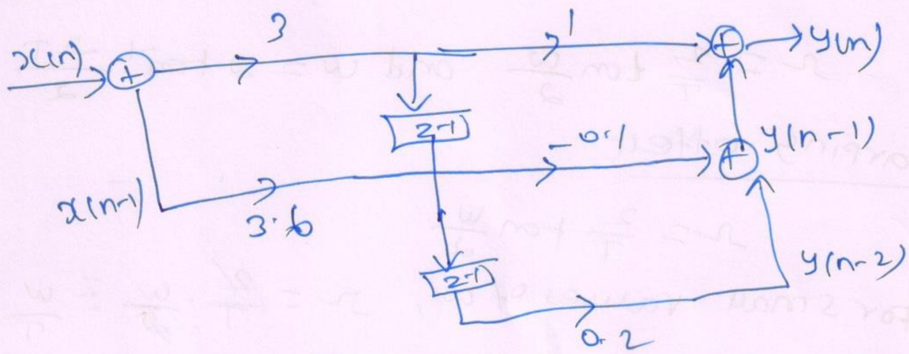
Let $3x(n) + 3.6x(n-1) = w(n)$

then $y(n) = -0.1\, y(n-1) + 0.2\, y(n-2) + w(n)$.

For direct form-I



for direct form II



8) By using bilinear transformation method.

1. From the given specifications, find prewarping analog frequencies using formula, $\Omega = \frac{2}{T}\tan\frac{\omega}{2}$,
2. using the analog frequencies find $H(s)$ of the analog filter
3. select the sampling rate of the digital filter call it T seconds per sample
4. sub $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ into the transfer function found in step 2

12. Solution

Given specifications.

$\alpha_p = 2dB$, $\Omega_p = 20 rad/sec$

$\alpha_s = 10dB$, $\Omega_s = 30 rad/sec$

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \frac{\Omega_s}{\Omega_p}}$$

$$\geq \frac{\log \sqrt{\frac{10-1}{10^{0.2}-1}}}{\log \left(\frac{30}{20}\right)} \geq 3.37$$

Rounding off N to the next highest integer

$N = 4$

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.84775s+1)}$$

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}} = 21.3868$$

$\therefore$ ~~H(s) =~~ sub $s \to \frac{s}{\Omega_c} = \frac{s}{21.3868}$ in H(s)

$$\therefore H(s) = \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 0.76537\left(\frac{s}{21.3868}\right)+1} \times \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 0.76537\left(\frac{s}{21.3868}\right)+1}$$

$$H(s) = \frac{0.20921 \times 10^6}{(s^2+16.3686s+457.394)(s^2+39.5176s+457.394)}$$

PART A

UNIT III FIR FILTERS

1. What are FIR filters?

The filter designed by selecting finite number of samples of impulse response (h(n) obtained from inverse fourier transform of desired frequency response H(w)) are called FIR filters

2. Write the steps involved in FIR filter design?

1. Choose the desired frequency response Hd(w)
2. Take the inverse fourier transform and obtain Hd(n)
3. Convert the infinite duration sequence Hd(n) to h(n)
4. Take Z transform of h(n) to get H(Z)

3. What are advantages of FIR filter? (University)

Linear phase FIR filter can be easily designed. Efficient realization of FIR filter exists as both recurrisive and non recursive structures. FIR filter realized non recursively are stable. The round off noise can be made small in non recursive realization of FIR filter

4. What are the disadvantages of FIR Filter? (University)

The duration of impulse response should be large to realize sharp cutoff filters. The non integral delay can lead to problems in some signal processing applications.

5. What is the necessary and sufficient condition for linear phase characteristic in FIR filter? (University)

The necessary and sufficient condition for linear phase characteristic in FIR filter is, the impulse response h(n) of the system should have the symmetry property i.e., H(n) = h(N-1-n)

where N is the duration of the sequence.

6. List the well known design technique for linear phase FIR filter design? (University)

1. Fourier series method and window method
2. Frequency sampling method.
3. Optimal filter design method.

7. Distinguish between FIR filters and IIR filters.

FIR filter	IIR filter
1. These filters can be easily designed to	These filters do not have linear phase.
2. FIR filters can be realized recursively and non-recursively.	IIR filters are easily realized recursively.
3. Greater flexibility to control the shape of their magnitude response.	Less flexibility, usually limited to specific kind of filters.
4. Errors due to round off noise are less severe in FIR filters, mainly because feedback is not used.	The round off noise in IIR filters is more.

8 What is Gibb's Phenomenon?(MAY/JUNE-12)

One possible way of finding an FIR filter that approximates H(ejw) would be to truncate the infinite Fourier series at n=±(N-1/2).Direct truncation of the series will lead to fixed percentage overshoots and undershoots before and after an approximated discontinuity in the frequency response.

9. List the steps involved in the design of FIR filters using windows.

1. For the desired frequency response Hd(w), find the impulse response hd(n) using Equation

$$h_d(n)=1/2\pi \int_{-\pi}^{\pi} H_d(w)e^{jwn}dw$$

2. Multiply the infinite impulse response with a chosen window sequence w(n) of length N to obtain filter coefficients h(n),i.e.,

$$h(n)= h_d(n)w(n) \text{ for } |n|\leq(N-1)/2$$
$$= 0 \quad \text{otherwise}$$

3. Find the transfer function of the realizable filter (N-1)/2

$$H(z)=z^{-(N-1)/2} [h(0)+\sum_{n=0}^{(N-1)/2} h(n)(z^n+z^{-n})]$$

10. What are the desirable characteristics of the window function?

The desirable characteristics of the window are

1. The central lobe of the frequency response of the window should contain most of the energy and should be narrow.
2. The highest side lobe level of the frequency response should be small.
3. The side lobes of the frequency response should decrease in energy rapidly as ω tends to Π.

11. Give the equations specifying the following windows.

a. Rectangular window:

The equation for Rectangular window is given by

$$W(n)= 1 \quad 0 \leq n \leq M-1$$
$$0 \quad \text{otherwise}$$

b. Hamming window:

The equation for Hamming window is given by

$$W_H(n)= 0.54-0.46 \cos 2\pi n/M-1 \quad 0 \leq n \leq M-1$$
$$0 \quad \text{otherwise}$$

c. Hanning window:

The equation for Hanning window is given by

$$W_{Hn}(n)= 0.5[1- \cos 2\pi n/M-1] \quad 0 \leq n \leq M-1$$
$$0 \quad \text{otherwise}$$

d. Bartlett window:

The equation for Bartlett window is given by

$$W_T(n)= 1-\frac{2|n-(M-1)/2|}{M-1} \quad 0 \leq n \leq M-1$$
$$0 \quad \text{otherwise}$$

e. Kaiser window:

The equation for Kaiser window is given by

$$W_k(n)= \frac{I_o[\alpha\sqrt{1-(2n/N-1)^2}]}{I_o(\alpha)} \quad \text{for } |n| \leq \frac{N-1}{2}$$
$$0 \quad \text{otherwise}$$

where α is an independent parameter.

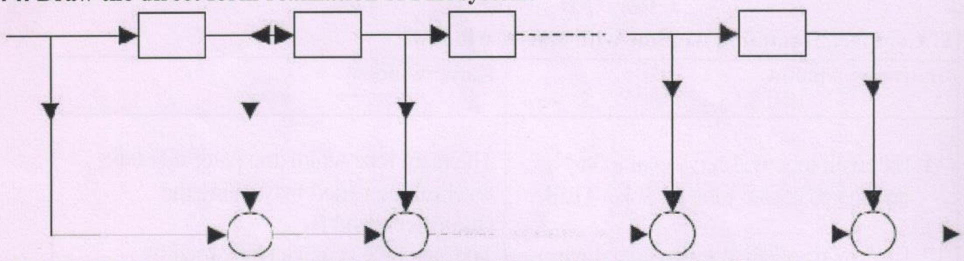
12. What are the advantages of Kaiser window?

- It provides flexibility for the designer to select the side lobe level and N
- It has the attractive property that the side lobe level can be varied continuously from the low value in the Blackman window to the high value in the rectangular window.

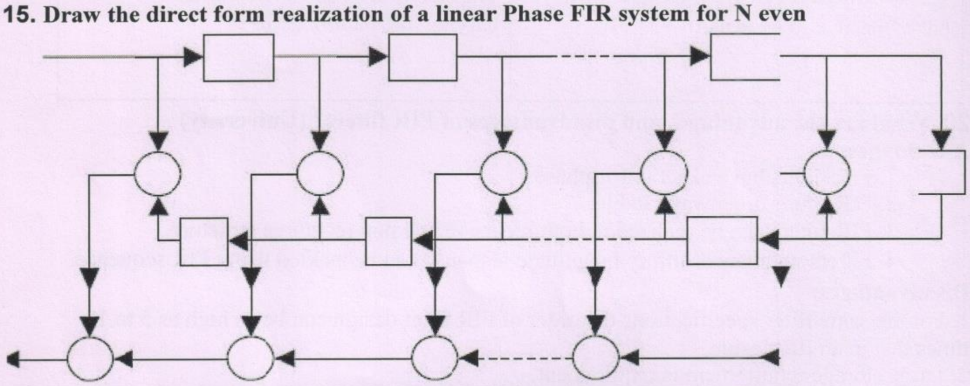
13. What is the principle of designing FIR filter using frequency sampling method?

In frequency sampling method the desired magnitude response is sampled and a linear phase response is specified .The samples of desired frequency response are identified as DFT coefficients. The filter coefficients are then determined as the IDFT of this set of samples.

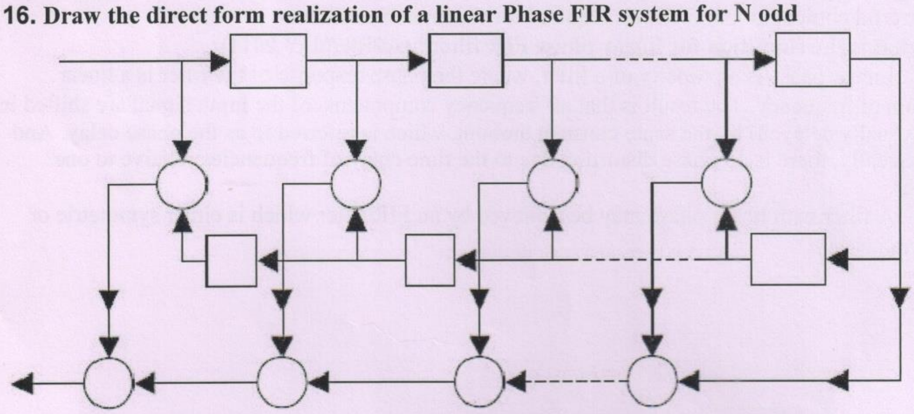
14. Draw the direct form realization of FIR system.



15. Draw the direct form realization of a linear Phase FIR system for N even



16. Draw the direct form realization of a linear Phase FIR system for N odd



17. State the equations used to convert the lattice filter coefficients to direct form FIR Filter coefficient.

$\alpha_m(0) = 1$

$\alpha_m(m) = k_m$

$\alpha_m(k) = \alpha_{m-1}(k) + \alpha_m(m) \cdot \alpha_{m-1}(m-k)$

18. State the equations used to convert the FIR filter coefficients to the lattice filter Coefficient.

For an M stage filter , $\alpha_{m-1}(0) = 1$ and $k_m = \alpha_m(m)$

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - \alpha_m(m) \cdot \alpha_m(m-k)}{1-\alpha_m^2(m)}, \quad 1 \le k \le m-1$$

19. Compare Hamming window with Kaiser window.

Hamming window	Kaiser window
1.The main lobe width is equal to $8\pi/N$ and the peak side lobe level is –41dB. 2.The low pass FIR filter designed will have first side lobe peak of –53 dB	The main lobe width ,the peak side lobe level can be varied by varying the parameter α and N. The side lobe peak can be varied by varying the parameter α.

20. What are the advantages and disadvantages of FIR filters? (University)

Advantages:

1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non recursive structure.
4. Filters with any arbitrary magnitude response can be tackled using FIR sequence.

Disadvantages:

1. For the same filter specifications the order of FIR filter design can be as high as 5 to 10 times that in an IIR design.
2. Large storage requirement is requirement
3. Powerful computational facilities required for the implementation.

21. What is the condition for linear phase FIR filter? (APRIL/MAY 2011)

Linear phase is a property of a filter, where the phase response of the filter is a linear function of frequency. The result is that all frequency components of the input signal are shifted in time (usually delayed) by the same constant amount, which is referred to as the phase delay. And consequently, there is no phase distortion due to the time delay of frequencies relative to one another.

A filter with linear phase may be achieved by an FIR filter which is either symmetric or antisymmetric.

PART - B

1. Design a high pass filter with a frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1 & for \quad \frac{\pi}{4} \le |\omega| \le \pi \\ 0 & for \quad |\omega| \le \frac{\pi}{4} \end{cases}$$

Find the values of $h(n)$ for N – 11 using hamming window. Find $H(z)$ and determine the magnitude response.

2. (i) Determine the coefficients $\{h(n)\}$ of a linear phase FIR filter of length M = 15 which has a symmetric unit sample response and a frequency response that satisfies the condition

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & for \quad k = 0,1,2,3 \\ 0 & for \quad k = 4,5,6,7 \end{cases}$$

(ii) Obtain the linear phase realization of the system function

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

3. Realize the system function by linear phase FIR structure

$$H(z) = \frac{2}{3}z + 1 + \frac{2}{3}z^{-1}$$

4. Explain the designing of FIR filters using windows?

5. Design an ideal high pass filter using hanning window with a frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1 & for \quad \frac{\pi}{4} \le |\omega| \le \pi \\ 0 & for \quad |\omega| \le \frac{\pi}{4} \end{cases}$$

Assume N = 11.

6. Design a FIR low pass filter having the following specifications using Hanning window

$$H_d(e^{j\omega}) = \begin{cases} 1 & for \quad -\frac{\pi}{6} \le |\omega| \le \frac{\pi}{6} \\ 0 & for \quad otherwise \end{cases}$$

Assume N – 7

7. Design an FIR low pass digital filter using the frequency sampling method for the following specifications

Cut off frequency = 1500Hz

Sampling frequency = 15000Hz

Order of the filter N = 10

Filter Length required L = N+1 = 11

8. (i) Explain with neat sketches the implementation of FIR filters in direct form and Lattice form

   (ii) Design a digital FIR band pass filter with lower cut off frequency 2000Hz and upper cut off frequency 3200 Hz using Hamming window of length N = 7. Sampling rate is 10000Hz.

9. (i)Determine the frequency response of FIR filter defined by

$$y(n) = 0.25x(n) + x(n-1) + 0.25x(n-2)$$

   (ii) Discuss the design procedure of FIR filter using frequency sampling method.

10. Design an FIR filter using hanning window with the following specification

$$H_d(e^{j\omega}) = \begin{cases} 1 & for \quad \frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ e^{-j2\omega} & for \quad \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

   Assume N = 5.

11. (i) Explain briefly how the zeros in FIR filter is located.

   (ii) Using a rectangular window technique, design a low pass filter with pass band gain of unity cut off frequency of 1000Hz and working at a sampling frequency of 5 kHz. The length of the impulse response should be 7.

12. Consider an FIR lattice filter with coefficients $k_1 = 1/2$; $k_2 = 1/3$; $k_3 = 1/4$. Determine the FIR filter coefficients for the direct form structure

3-1

1) Solution

For Hamming window $\quad h_d(n) = \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{n\pi}{4}\right]$

$$w_H(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \text{ for } -(N-1)/2 \le n \le (N-1)/2$$
$$0 \quad \text{otherwise.}$$

$$h(n) = h_d(n) \cdot w_H(n) \text{ for } -5 \le n \le 5$$
$$0 \quad \text{otherwise}$$

$h(0) = 0.25$, $h(1) = h(-1) = -0.2052$, $h(2) = h(-2) = -0.1084$

$h(3) = h(-3) = -0.03$, $h(4) = h(-4) = 0$, $h(5) = 0.0036$

$$H(z) = h(0) + \sum_{n=1}^{5} [h(n)(z^{-n} + z^{n})]$$

$$= 0.25 - 0.2052(z^{-1} + z) - 0.1084(z^{-2} + z^{2}) - 0.03(z^{-3} + z^{3}) + 0.0036(z^{-5} + z^{5})$$

$H'(z) = z^{-5} H(z)$.

The filter Co-efficients of Causal filter are,

$h(0) = h(10) = 0.0036$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.033$

$h(3) = h(7) = -0.1084$

$h(4) = h(6) = -0.2052$

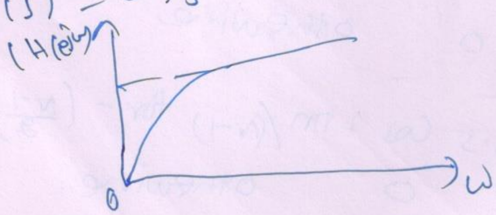
$h(5) = 0.25$

$\bar{H}(e^{jw}) = \sum_{n=0}^{5} a(n) \cos wn.$

$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.25$

$\bar{H}(e^{jw}) = 0.25 - 0.4104 \cos w - 0.2168 \cos 2w - 0.06 \cos 3w + 0.0072 \cos 5w$

$a(n) = 2h\left[\frac{N-1}{2} - n\right]$

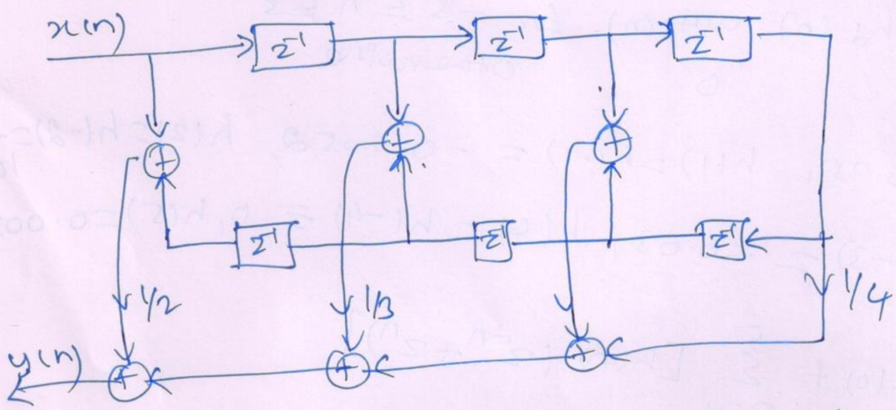


2) Linear Phase FIR filters:

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

Solution

$$h(n) = h(N-1-n)$$



3) 5)(b). Designing of FIR filters using window:

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n}$$

where,
$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$n = \pm\left(\frac{N-1}{2}\right)$$

$$h(n) = \begin{cases} h_d(n)\, w(n) & \text{for all } |n| \le \left(\frac{N-1}{2}\right) \\ 0 & \text{for } |n| > \left(\frac{N-1}{2}\right) \end{cases}$$

For Rectangular window

$$w_R(n) = \begin{cases} 1 & \text{for } -(N-1)/2 \le n \le (N-1)/2 \\ 0 & \text{otherwise} \end{cases}$$

For Hanning window

$$w_{Hn}(n) = \begin{cases} 0.5 + 0.5\cos 2\pi n/(N-1) & \text{for } -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & \text{otherwise.} \end{cases}$$

For Hamming window

$$w_H(n) = 0.46 + 0.54\cos\left(\frac{2\pi n}{N-1}\right), \text{ for } -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right)$$
$$0 \quad \text{otherwise}$$

The filter co-efficients using windowing techniques.

$$h(n) = h_d(n)\, w(n) \quad \text{for } -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right)$$
$$0 \quad \text{otherwise}$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\left(\frac{N-1}{2}\right)} h(n)\left[z^{-n} + z^{n}\right]$$

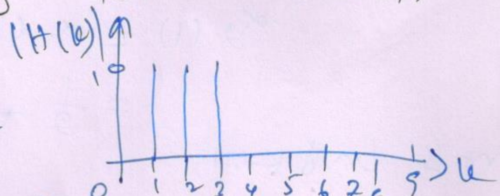
The transfer function of the realizable filter is

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

Determine the co-efficients $\{h(n)\}$ of a linear phase FIR filter of length $M=15$ which has a symmetric unit sample response and a frequency response that satisfies the condition

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & \text{for } k = 0,1,2,3 \ \& \ 12 \le k \le 14 \\ 0 & \text{for } k = 4,5,6,7 \end{cases}$$

Solution



$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$$
$$= \left(\frac{-14}{15}\right)\pi k, \ 0 \le k \le 7$$

$$H(k) = e^{-j14\pi k/15} \quad \text{for } k = 0,1,2,3$$
$$0 \quad \text{for } 4 \le k \le 11.$$
$$= e^{-j14\pi(k-15)/15} \quad \text{for } 12 \le k \le 14$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)\, e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} Re\left(e^{-j14\pi k/15}\, e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} \cos\left(\frac{2\pi k(7-n)}{15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

$h(0) = h(14) = -0.05$, $h(1) = h(13) = 0.041$, $h(2) = h(12) = 0.0666$

$h(3) = h(11) = -0.0365$, $h(4) = h(10) = -0.1078$, $h(5) =$

$h(9) = 0.034$, $h(6) = h(8) = 0.3188$, $h(7) = 0.466$.

12) FIR lattice filter with co-efficients.

$k_1 = 1/2$, $k_2 = 1/3$, $k_3 = 1/4$.

Solution

$\alpha_3(0) = 1$, $\alpha_3(3) = k_3 = 1/4$

$\alpha_1(1) = k_1 = \frac{1}{2}$; $\alpha_2(2) = k_2 = 1/3$

$$\alpha_m(k) = \alpha_{m-1}(k) + k_m\, \alpha_{m-1}(m-k)$$

for $m=2$ and $k=1$

$$\alpha_2(1) = \alpha_1(1) + k_2\alpha_1(1)$$

$$= \frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2} = \frac{3+1}{6} = 2/3$$

for $m=3$ and $k=1$

$$\alpha_3(1) = \alpha_2(1) + \alpha_3(3)\,\alpha_2(2)$$

$$= \frac{2}{3} + \frac{1}{4}\cdot\frac{1}{3} = \frac{8+1}{12} = \frac{3}{4}$$

for $m=2$ and $k=2$

$$\alpha_3(2) = \alpha_2(2) + \alpha_3(3)\,\alpha_2(1)$$

$\alpha_3(0) = 1$, $\alpha_3(1) = \frac{3}{4}$, $\alpha_3(2) = \frac{1}{2}$, $\alpha_3(3) = 1/4$

$$= \frac{1}{3} + \frac{1}{4}\cdot\frac{2}{3} = \frac{4+2}{12} = 1/2$$

PART A

UNIT IV FINITE WORD LENGTH EFFECTS

1. What are the different types of arithmetic in digital systems?
There are three types of arithmetic used in digital systems. They are

- fixed point arithmetic
- floating point
- block floating point arithmetic

2. What is meant by fixed point number? (University)
In fixed point number the position of a binary point is fixed. The bit to the right represent the fractional part and those to the left is integer part.

3. What are the different types of fixed point arithmetic?
Depending on the negative numbers are represented there are three forms of fixed point arithmetic. They are

- sign magnitude
- 1's complement
- 2's complement

4. What is meant by sign magnitude representation?
For sign magnitude representation the leading binary digit is used to represent the sign. If it is equal to 1 the number is negative, otherwise it is positive.

5. What is meant by 1's complement form?
In 1,s complement form the positive number is represented as in the sign magnitude form. To obtain the negative of the positive number, complement all the bits of the positive number.

6. What is meant by 2's complement form?
In 2's complement form the positive number is represented as in the sign magnitude form. To obtain the negative of the positive number, complement all the bits of the positive number and add 1 to the LSB.

7. What is meant by floating point representation? (University)
In floating point form the positive number is represented as F =2CM,
where M is mantissa, is a fraction such that1/2<M<1and
C is the exponent can be either positive or negative.

8. What are the advantages of floating point representation?
1. Large dynamic range
2. Overflow is unlikely.

9. What are the quantization errors due to finite word length registers in digital filters?(University)
1. Input quantization errors
2. Coefficient quantization errors
3. Product quantization errors

10. What is input quantization error?
The filter coefficients are computed to infinite precision in theory. But in digital computation the filter coefficients are represented in binary and are stored in registers. If a b bit register is used the filter coefficients must be rounded or truncated to b bits, which produces an error.

11. What is product quantization error?
The product quantization errors arise at the output of the multiplier. Multiplication of a b bit data with a b bit coefficient results a product having 2b bits. Since a b bit register is used the multiplier output will be rounded or truncated to b a bit which produces the error.

12. What are the different quantization methods? (University)

1. Truncation
2. Rounding

13. What is truncation? (University)

Truncation is a process of discarding all bits less significant than LSB that is retained

14. What is rounding? (University)

Rounding a number to b bits is accomplished by choosing a rounded result as the b bit number closest number being unrounded.

15. What are the two types of limit cycle behavior of DSP?

1. Zero limit cycle behavior
2. Over flow limit cycle behavior

16. What are the methods to prevent overflow?

1. Saturation arithmetic and
2. Scaling

PART – B

1. Discuss in detail the errors resulting from rounding and truncation?
2. (i) Explain the limit cycle oscillations due to product round off and overflow errors?

   (ii) Explain how reduction of product round-off error is achieved in digital filters?
3. (i) Explain the effects of co-efficient quantization in FIR filters?

   (ii) Distinguish between fixed point and floating point arithmetic
4. With respect to finite word length effects in digital filters, with examples discuss about

   (i) Over flow limit cycle oscillation

   (ii) Signal scaling
5. Consider a second order IIR filter with

$$H(z) = \frac{1.0}{(1-0.5Z^{-1})(1-0.45Z^{-1})}.$$

   Find the effect on quantization on pole locations of the given system function in direct form and in cascade form. Assume $b = 3$ bits.
6. What is called quantization noise? Derive the expression for quantization noise power.
7. (i) Compare the truncation and rounding errors using fixed point and floating point representation.

   (ii) Represent the following numbers in floating point format with five bits for mantissa and three bits for exponent.

   (a) 7_{10}

   (b) 0.25_{10}

   (c) -7_{10}

   (d) -0.25_{10}
8. Determine the dead band of the system $y(n) = 0.2y(n-1) + 0.5y(n-2) + x(n)$

   Assume 8 bits are used for signal representation.
9. (a) i) Explain the characteristics of limit cycle oscillation with respect to the systen described by the difference equation : y(n) = 0.95 y(n-1) + x (n) ; x(n)= 0 and y(n-1)= 13. Determine the dead range of the system.

   ii) Explain the effects of coefficient quantization in FIR filters.
10. i) Derive the signal to quantization noise ratio of A/D converter.

    ii) Compare the truncation and rounding errors using fixed point and floating point representation. ‘

1) Error due to truncation and Rounding

Type of Quantization	Type of arithmetic	Fixed-point number range	Floating point number range
Rounding	sign-magnitude, one's, two's	$-\frac{2^{-b}}{2} \le e \le \frac{2^{-b}}{2}$	$-2^{-b} \le \varepsilon < 2^{-b}$
Truncation	2's	$-2^{-b} < e \le 0$	$-2.2^{-b} < \varepsilon \le 0,\ m>0$; $0 \le \varepsilon < 2.2^{-b},\ m<0$
Sign-magnitude truncation	1's, 2's	$-2^{-b} < e \le 0,\ x>0$; $0 \le e < 2^{-b},\ x<0$	$-2.2^{-b} < \varepsilon \le 0$

2) Product Quantization error:

$$x_q(n) \xrightarrow{a} a\,x_q(n) \longrightarrow \oplus \xleftarrow{} e(n) \longrightarrow y(n) = a\,x_q(n) + e(n)$$

$$\sigma_{ok}^2 = \sigma_e^2 \frac{1}{2\pi j} \oint_C H_k(z)\, H_k(z^{-1})\, z^{-1}\, dz$$

3) over flow limit cycle oscillation.

In addition to limit cycle oscillations caused by rounding the result of multiplication, there are several types of limit cycle oscillation caused by addition, which make the filter output oscillation between maximum and minimum amplitude. Such limit cycle oscillations are overflow oscillation.

4) Signal Scaling:-

$$S_0^2 = \frac{1}{\frac{1}{2\pi j}\oint_C S(z)\, S(z^{-1})\, z^{-1}\, dz} = \frac{1}{\frac{1}{2\pi j}\oint_C \frac{z^{-1}\, dz}{D(z)\, D(z^{-1})}}$$

$$S_0^2 = \frac{1}{I} \quad \text{where} \quad I = \frac{1}{2\pi j}\oint_C \frac{z^{-1}\, dz}{D(z)\, D(z^{-1})}$$

5).
$$H(z) = \frac{1.0}{(1-0.5z^{-1})(1-0.45z^{-1})}$$

$e_1(n)$ is $H(z)$
$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.45z^{-1})}$$

and $e_2(n)$ is $H_2(z) = \frac{1}{(1-0.45z^{-1})}$

$$\sigma_{01}^2 = \frac{1}{2\pi j}\oint_c H(z)\,H(z^{-1})\,z^{-1}dz$$

For
$$= \sigma_e^2 \frac{1}{2\pi j}\oint_c \frac{1}{1-0.5z^{-1}} \cdot \frac{1}{1-0.45z^{-1}} \cdot \frac{1}{1-0.5z} \cdot \frac{1}{1-0.45z}\, z^{-1}dz$$

$$\sigma_0^2 = \sigma_{01}^2 + \sigma_{02}^2$$

$$\boxed{\sigma_0^2 = \frac{2^{-2b}}{12}(5.4315)}$$

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.375z^{-1})}$$

6) Quantization noise

$x(t) \rightarrow$ [Sample] $\xrightarrow{x(n)}$ [Quantize] $\rightarrow x_q(n)$

$$q = \frac{2}{2^{b+1}} = 2^{-b}$$

If $b = 3$ bits then $q = 2^{-3} = 0.125$

The common methods of quantization are
1. Truncation
2. Rounding.

1) Input quantization error
2) product quantization error
3) co-efficient quantization error

PART A

UNIT V MULTIRATE SIGNAL PROCESSING

1. State some applications of DSP?

1. Speech processing
2. Image processing
3. Radar signal processing.

2. Define sampling rate conversion.

Sampling rate conversion is the process of converting the sequence x(n) which is got from sampling the continuous time signal x(t) with a period T, to another sequence y(k) obtained from sampling x(t) with another period T'.

3. State the methods to convert the sampling rate.

There are two methods:

- Resampling after reconstruction
- Conversion in digital domain

4. What is multirate signal processing?

Multirate signal processing is the technique of processing the signal with multiple sampling rates.

Advantages:

- Computational complexity is less
- Finite arithmetic effects are less
- Filter order required are low
- Sensitivity to filter coefficient lengths is less

5. State the applications of multirate signal processing.

- Sub-band coding
- Voice privacy using analog phone lines
- Signal compression by subsampling
- A/D and D/A convertors

The above applications come under the areas given below:

- Communication Systems
- Speech and audio processing systems
- Antenna systems
- Radar Systems

6. What is decimation?

Decimation is the process of reducing the sampling rate of the signal. It is otherwise called down-sampling or sampling rate compression.

7. What is interpolation?

Interpolation is the process of increasing the sampling rate of the signal. It is otherwise called down-sampling or sampling rate expansion.

8. Give short note on sub-band coding?

signals which occupy contiguous frequency bands analysis filter bank. These signals are down-sampled, yielding sub- band signals, which are then compressed using encoders. The compressed signals are multiplexed and transmitted. On the receiving side, reverse operations are carried out. This process yields better compression ratio, because each sub-band signal can be represented using a different number of bits.

9. Give brief not on Speech Processing.

Speech processing includes processing like encoding, synthesis and recognition.

Encoding is performed to remove the redundant signal in a speech signal.

Compression/coding is performed in transmitter side and thus **synthesis** is required in receiver side.

Recognition is used to recognize both the speech and the speaker.

10. What are the different techniques of voice compression and coding?

- Waveform coding
- Transform Coding
- Frequency band encoding
- Parametric methods

11. Give short notes on image enhancement.

Image enhancement focuses mainly on the features of an image. The various feature enhancements are sharpening the image, edge enhancement, filtering, contrast enhancement, etc.

12. Give short notes on adaptive filters.

Adaptive filters are linear filters used in various areas where the statistical knowledge of the signals to be filtered/analyzed are not known a priori or the signals may be slowly time variant. Both IIR and FIR filters can be used in adaptive filtering, but FIR filters are mostly used due to its simplicity and adjustable zeros.

13. State the applications of adaptive filtering.

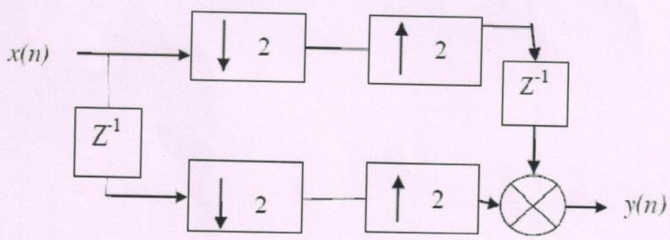
- Adaptive noise cancelling
- Line Enhancing
- Frequency Tracking
- Channel Equalization
- Echo cancellation

PART – B

1. Explain the poly phase structure of decimator and interpolator?(Nov2010)
2. Discuss the procedure to implement digital filter bank using multi rate signal processing? (Nov2010)
3. (i) Explain how various sound effects can be generated with the help of DSP?

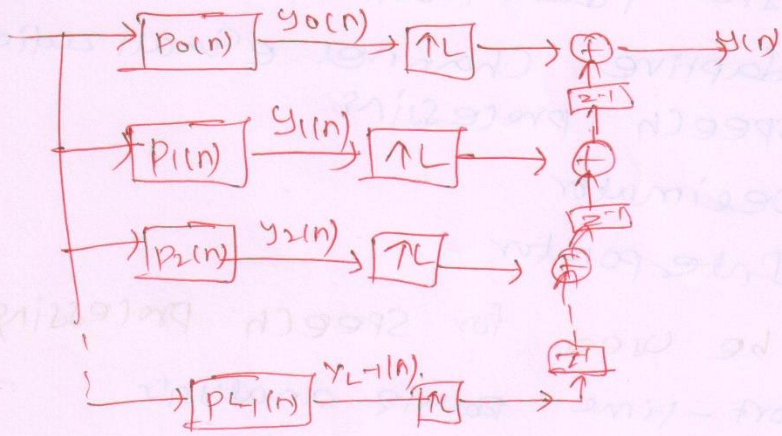
   (ii) State the applications of multirate signal processing? (May2011)
4. (i) Explain how DSP can be used for speech processing?

   (ii) Explain in detail about decimation and interpolation? (May2011)
5. For the multi rate system shown in figure, find the relation between $x(n)$ and $y(n)$ (Nov2011)

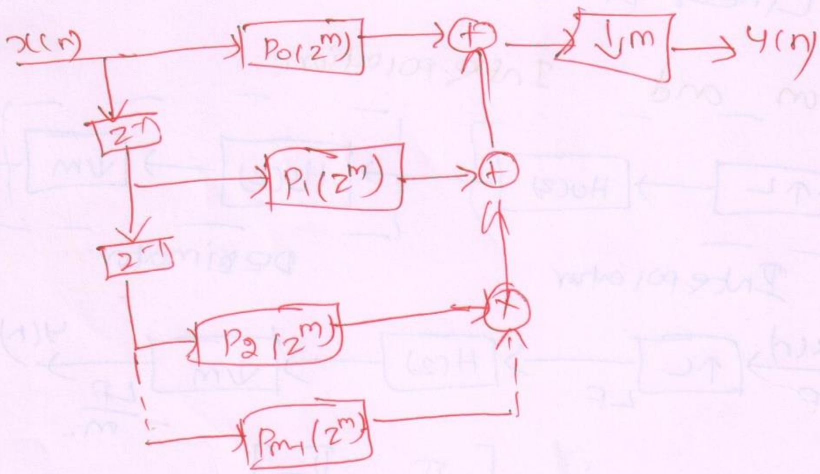


6. Explain the efficient transversal structure for decimator and interpolator? (Nov2011)
7. Explain sub band coding in detail (May2012)
8. Explain sampling rate conversion by a rational factor and derive input and output relation in both time and frequency domain (Nov2012)
9. Explain the design of narrow band filter using sampling rate conversion(Nov2012)
10. Explain the design steps involved in the implementation of multistage sampling rate converter. (Nov2013)
11. Explain the implementation steps in speech coding using transform coding?(Nov2013)
12. A signal $x(n)$ is given by $x(n) = \{0,1,2,3,4,5,6,0,1,2,3....\}$
    (i) Obtain the decimated signal with a factor of 2.
    (ii) Obtain the interpolated signal with a factor of 2. (May2013)

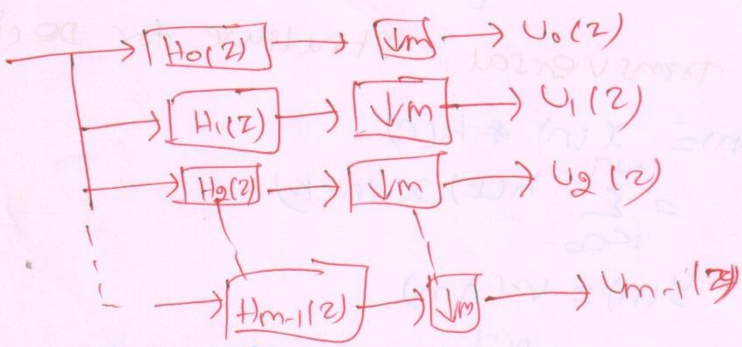
1) Polyphase Structure of Interpolator:



Polyphase Structure of Decimator:-



2) Digital filter banks:-



*) Analysis filter bank

*) synthesis filter bank

*) Sub band coding filter bank

*) Quadrature - mirror filter bank

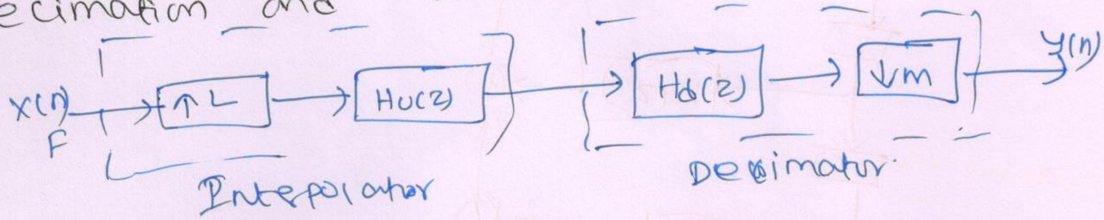
3) application of multirate Signal processing.

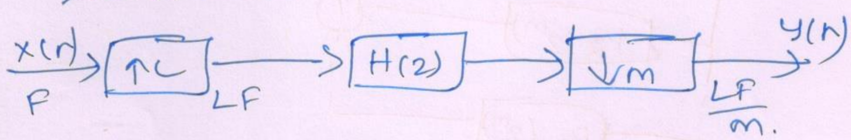
1. System identification
2. Adaptive Channel equalization.
3. speech processing.
4. Decimator
5. Interpolator.

4) DSP can be used for speech processing:

1. Short-time Fourie analysis
2. cepstral analysis
3. Linear predition analysis.

Decimation and Interpolation:

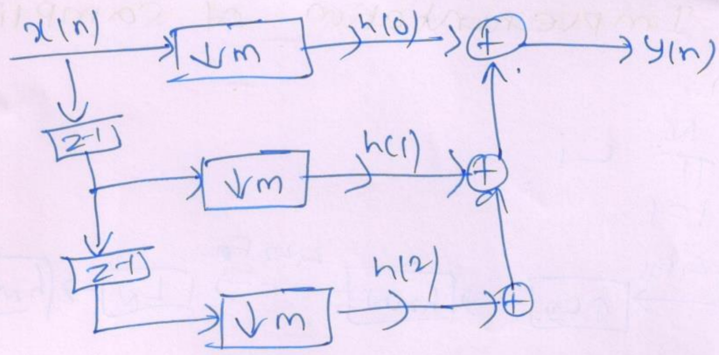




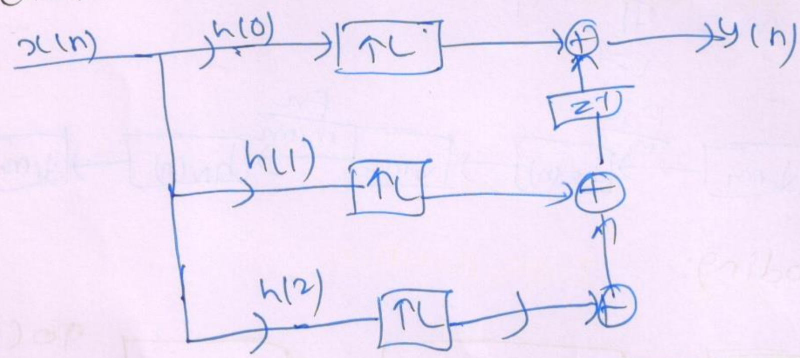
$$w_c = \min\left[\frac{\pi}{L}, \frac{\pi}{m}\right]$$

Efficient transversal Structure for Decimator:

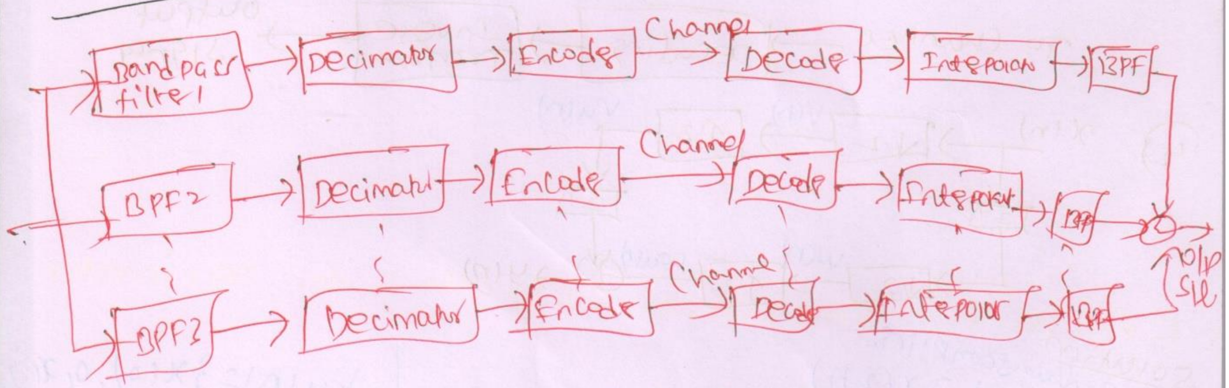
$$v(n) = x(n) * h(n)$$
$$= \sum_{k=0}^{N-1} h(k)\, x(n-k)$$
$$y(n) = v(nm)$$
$$= \sum_{k=0}^{N-1} h(k)\, x(nm-k)$$



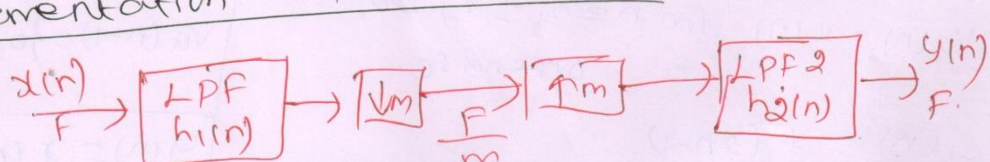
Efficient Transversal Structure for Interpolator



Subband Coding:



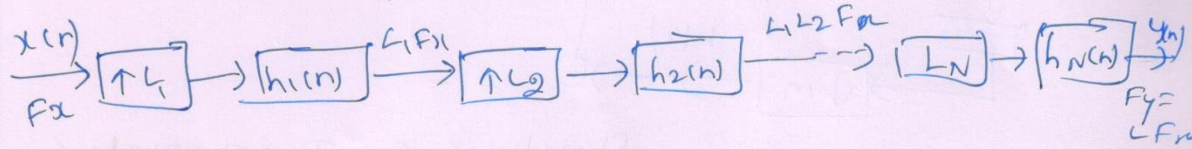
Implementation of Narrow band



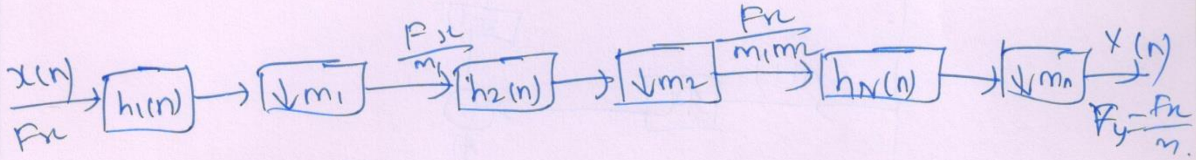
The filters h1(n) and h2(n) in the decimator and interpolator are low pass filters.

10) Multi stage Implementation of sampling rate conversion:-

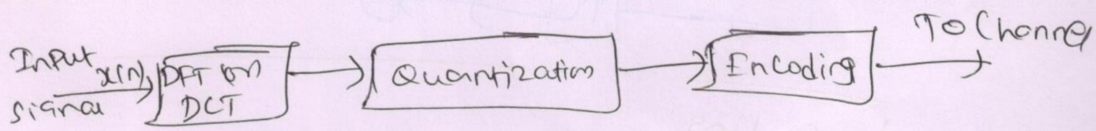
$$L = \prod_{i=1}^{N} L_i$$

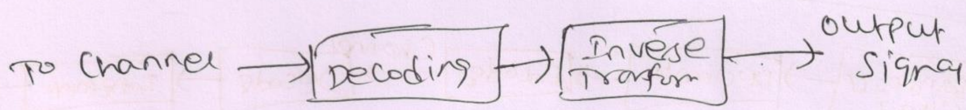


$$M = \prod_{i=1}^{N} M_i$$

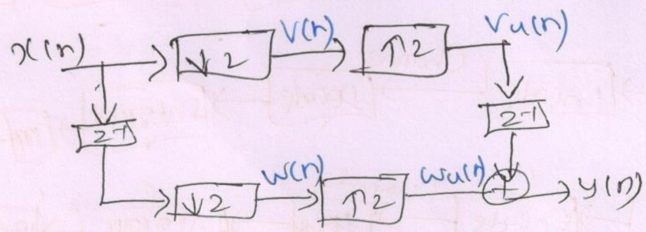


11) Speech Coding:-





12)



Solution

after downsampling $U(n) = x(2n)$

$Vu(n) = x(n)$ for $n = 0, \pm 2, \pm 4.$

$\quad 0$ otherwise.

$w(n) = x(2n-1)$

$wu(n) = x(n-1)$ for $n = 0, \pm 2, \pm 4.$

$\quad 0$ otherwise

$y(n) = Vu(n-1) + wu(n)$

$Vu(n) = \{x(0), 0, x(2), 0, \ldots\}$

$Vu(n-1) = \{0, x(0), 0, x(2), 0 \ldots\}$

$$\boxed{y(n) = x(n-1)}$$